Nonlinear response of plain concrete shear walls with damage

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ABSTRACT | During earthquakes, the natural frequencies of concrete structures are often significantly lower than those predicted by conventional linear elasticity. This can cause severe motion of piping systems because the design of support structures utilizes values of predicted natural frequencies. In an experimental program to investigate the problem, a further decrease in stiffness was noted for model concrete structures. In this study continuum damage mechanics is proposed as a constitutive model for describing both the changes in natural frequencies, and the reduction in initial stiffness of small concrete structures. Structural members made with brittle materials such as concrete experience damage under seismic excitation, which is reflected through altered natural frequencies for the structure. With regard to scale models, it is suggested that microcracking as a result of shrinkage may be the source of the loss in initial stiffness. Shrinkage cracks are easily reflected in the constitutive equation as initial isotropic damage. Finite element predictions based on anisotropic damage mechanics indicate that the proposed approach may be practicable for routine engineering analyses.

1 Introduction

The design of piping support systems is based in part on the natural frequencies of the primary structure so that potential motion of the pipes can be minimized if the structure is subjected to earthquakes. For certain nuclear power plant structures made of reinforced concrete some data suggest that natural frequencies displayed during seismic disturbances are significantly lower than those computed based on linear elasticity which is considered appropriate because of the small amplitudes that are experienced. To address this apparent anomaly, Endebrock et al. [1] at Los Alamos National Laboratory conducted a program to determine if such an effect could be experimentally demonstrated.

Shear walls are important structural members, which resist horizontal forces due to wind or seismic motion and can be designed to support gravity loads as well. Since the full-size construction and testing of shear walls are expensive, many studies have been performed on scaled down models. Endebrock et al. [1] chose to test both scaled models and prototype walls. Their results did indeed show that if the model shear wall is loaded to a level that might be experienced during the beginning phase of an earthquake ground excitation then there is a reduction in the values of natural frequencies.

However, the conclusion was clouded by other observations. First, the models exhibited nonlinear behavior even at relatively small loads, as measured values of fundamental frequencies were lower than those predicted by linear elasticity. The elasticity parameters were obtained in the traditional manner of testing cylindrical specimens. Second, with the use of appropriate scaling laws, the experimental values of natural

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frequencies obtained from the model walls were lower than those obtained from prototype walls.

The basis of this work is the proposition that the relatively new field of continuum damage mechanics might be useful to help explain these observed features. The following assumptions are made: (1) when concrete is loaded, even to relatively low levels, damage occurs and this damage is most easily noticed in subsequent load cycles such as those occurring from sinusoidal base excitation, and (2) shrinkage cracks, which could be interpreted as initial damage, is more significant for structural elements with large surface-to-volume ratios than for those with small ratios so that model shear walls should exhibit a lower initial stiffness than the prototype shear wall formed from the same batch of concrete.

The scope of the research is limited to a plausibility study because continuum damage mechanics was, and remains to a considerable degree, an unknown field to the engineering community. Also, an attempt at a detailed correlation would have required the incorporation of a great amount of construction detail such as steel reinforcement and the three-dimensional headwork attached to the shear wall which transmitted the load from the shaking device.

The source of the nonlinear behavior might be attributed to two distinct microstructural changes. One is the development of plastic flow along preferred dislocation planes that occurs under high confining pressures. The second is the nucleation and propagation of microcracks and microvoids. Since most conventional concrete structural members, including shear walls, are designed to perform under no or low confining pressure, it is plausible to assume that the second mechanism is the dominant one in this case. Continuum damage mechanics is the specific theory that addresses the effects of microcracks and microvoids on the material response, so that such an approach seems to be particularly appropriate.

The text describes the damage model used for the study. The basis for shrinkage damage is given together with a technique for incorporating shrinkage in the constitutive equation. Numerical results that must be considered primarily qualitative are shown to indicate that continuum damage mechanics is a powerful approach for addressing a significant engineering problem.

2 Damage mechanics

Continuum Damage Mechanics (CDM), which was first introduced by Kachanov [2], has attracted the attention of researchers in the past fifteen years. In particular, many advances have been made in the application of CDM to the brittle-fracturing processes and materials (Chen and Schreyer [3], Ju et al. [4], Karnawat and Yazdani [5]; Kracinovic [6], Krajcinovic et al. [7], Ortiz [8], Ortiz and Giannakopoulos [9], Schreyer and Neilson [10], Simo and Ju [11], and Stevens and Liu [12]). In this section a formulation (Yazdani and Schreyer [13]) is proposed that is consistent with the laws of thermodynamics and that utilizes experimental information concerning the modes of evolution of microcracks. If **E** and **e** denote the elasticity and strain tensors, respectively, the stored energy function for linear elasticity is defined to be

$$\Psi(\mathbf{e},\mathbf{E})=\frac{1}{2}\mathbf{e}:\mathbf{E}:\mathbf{e} \qquad (1)$$

where ":" indicates tensor contraction operation. It will be assumed that damage is reflected through the elasticity tensor, which is, therefore, listed as a variable. Suppose further that the internal energy, U, can be given in an uncoupled form as:

$$\mathrm{U}(\mathbf{e},\mathbf{E},\varepsilon)=\frac{1}{\rho}[\Psi(\mathbf{e},\mathbf{E})+\mathrm{U}_{\varepsilon}(\varepsilon)] \qquad (2)$$

in which ε denotes the entropy and ρ is the mass density. The stress tensor, **S**, the temperature, θ, and the conjugate thermodynamical force, **Q**, associated with **E** (Chaboche [14]) are given by the constitutive relations as

$$\mathbf{S}=\rho\frac{\partial U}{\partial \mathbf{e}}=\frac{\partial \psi}{\partial \mathbf{e}}=\mathbf{E}:\mathbf{e} \quad \text{(3a)}$$

$$\theta=\rho\frac{\partial U}{\partial \varepsilon}=\frac{\partial U_{\varepsilon}}{\partial \varepsilon} \quad \text{(3b)}$$

$$\mathbf{Q}=\rho\frac{\partial U}{\partial \mathbf{E}}=\frac{\partial \psi}{\partial \mathbf{E}}=\frac{1}{2}\mathbf{e}\otimes\mathbf{e} \quad \text{(3c)}$$

where the symbol "⊗" is used to denote tensor product operation. Define the dissipation to be

$$D_s=-\rho\frac{\partial U}{\partial \mathbf{E}}::\dot{\mathbf{E}}=-\mathbf{Q}::\dot{\mathbf{E}} \quad \text{(4)}$$

in which superposed dots denote derivatives with respect to time and "::" signifies a double tensor contraction. In the absence of a heat source and a heat flux, the first and second laws of thermodynamics reduce to

$$\theta\dot{\varepsilon}=D_s, \text{ and } \quad D_s\geq 0 \quad \text{(5)}$$

since the absolute temperature is positive. The flexibility tensor, **F**, is the inverse of the elasticity tensor, **E**, so that

$$\mathbf{F}:\mathbf{E}=\mathbf{I}, \qquad \dot{\mathbf{F}}:\mathbf{E}+\mathbf{F}:\dot{\mathbf{E}}=\mathbf{0}, \text{ and } \quad \dot{\mathbf{E}}=-\mathbf{E}:\dot{\mathbf{F}}:\mathbf{E} \quad \text{(6)}$$

where **I** denotes the fourth-order identity tensor. Let ω be a monotonically increasing variable introduced to parameterize the damage process which is described through the damage response tensor, **R**, as

$$\dot{\mathbf{F}}=\dot{\omega}\mathbf{R} \quad \text{(7)}$$

Substituting Equations (6) and (7) back in Equation (4), yields the following form of the dissipation:

$$D_s=\frac{1}{2}\mathbf{S}:\dot{\mathbf{F}}:\mathbf{S}=\frac{1}{2}\dot{\omega}\mathbf{S}:\mathbf{R}:\mathbf{S}\geq 0 \quad \text{(8)}$$

where the inequality condition will be satisfied if **R** is a positive, semi-definite tensor.

To progress further, a damage function Φ(S,R,ω) is introduced such that, if damage is occurring, the stress state is said to be on the damage surface, $\Phi = 0$, and if no damage is occurring, $\Phi < 0$. The condition $\Phi > 0$ is not allowed. To construct such a function, utilize the condition that the coefficient of $\dot{\omega}$ be positive during the damage process, i.e., let

$$\Phi=\frac{1}{2}\mathbf{S}:\mathbf{R}:\mathbf{S}-g^2=0 \quad \text{(9)}$$

where the function "g" can be given directly as a hardening and softening damage function, or g can be prescribed through a separate evolution equation:

$$\dot{g}=\dot{\omega}G(\mathbf{S},\mathbf{R},\omega) \quad \text{(10)}$$

for some function G.

The key part of the model is reflected through the tensor, **R**, which appears in the evolution equation for the flexibility (Equation 7). Figure 1 shows the two damage modes that are considered essential for describing the failure of concrete. Mode I refers to the cleavage type of cracking that occurs under tension, and mode II refers to a more complicated mechanism suggested by Horri and Nemat-Nasser [15] that involves shear sliding of an existing inclined flaw with tensile crack opening under a far-field compressive stress state. To reflect these two modes in the formulation, **R** is considered to be the linear combination of modes I and II response tensors as

$$\mathbf{R}=\mathbf{R}_{I}+\mathbf{R}_{II} \quad \text{(11)}$$

To develop the operators in (11), suppose the stress tensor is separated into positive and negative cones as $\mathbf{S}=\mathbf{S}^{+}+\mathbf{S}^{-}$ where the positive (negative) cone is defined to be the part of the stress tensor associated with positive (negative) eigenvalues of the stress tensor **S**. Following Ortiz [8], $\mathbf{R}_I$ is proposed to be:

$$\mathbf{R}_{I}=\frac{\mathbf{S}^{+}\otimes\mathbf{S}^{+}}{\mathbf{S}^{+}:\mathbf{S}^{+}} \quad \text{(12)}$$

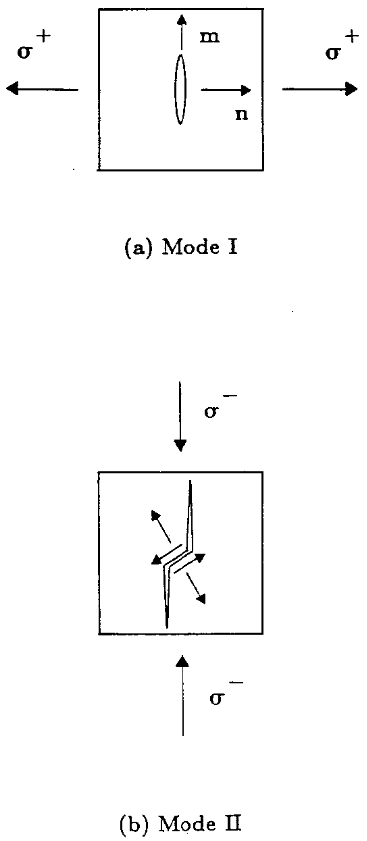

Figure 1.

A similar form is not adequate for mode II because damage is generally not observed under hydrostatic compression. The form suggested by Yazdani and Schreyer [13] is the following:

$$\mathbf{R}_{II} = \frac{\hat{\mathbf{S}}_s \otimes \hat{\mathbf{S}}_s}{\hat{\mathbf{S}}_s : \hat{\mathbf{S}}_s} + \alpha H[-\lambda_s][\mathbf{I} - \mathbf{i} \otimes \mathbf{i}] \qquad (13)$$

in which λ_s denotes the minimum eigenvalue of the shifted stress tensor, $\hat{S}_s$

$$\mathbf{S}_s = \mathbf{S}^- - \lambda\,\mathbf{i} \qquad (14)$$

and λ denotes the maximum (nonzero) eigenvalue of S^-. Here, **i** represents the second order identity tensor, and the Heaviside function H[·] is used in the second part of R_{II} to ensure that this term is activated only in compression. The material parameter, α, is determined from experimental data. This formulation implies that damage for compression occurs in the direction associated with the minimum eigenvalue (maximum absolute value) of S^- such as that reflected by vertical splitting under uniaxial compression. No damage is predicted for hydrostatic compression.

The damage function, g, which may also be considered the critical stress (Ortiz [8]) is determined from a uniaxial tensile or compression tests. An exponential function proposed by Smith and Young [16] is adapted to the present situation. Let f_t and f_c be limit stresses in uniaxial tension and compression, respectively, and let the initial modulus of elasticity be given by E_0. Guided by the work of Yazdani [17] and the experimental work by Smith and Young [16] the damage function is given as

$$g(S,\omega) = A(S)\; e\; \frac{\ln(1+E_0\omega)}{1+E_0\omega} \qquad (15)$$

where, "e" is the natural number in Equation (15) and A(**S**) denotes the maximum of the damage function, g, proposed by Yazdani [17] as

$$A^2(S) = \frac{(1+\xi^2+2\alpha\xi)f_t^2}{1+\xi^2(\frac{f_t}{f_c})^2+2\alpha\xi(\frac{f_t}{f_c})^2} \qquad (16)$$

in which ξ is given by

$$\xi = \left|\frac{tr(\mathbf{S}^-)}{tr(\mathbf{S}^+)}\right| \qquad (17)$$

If ξ is zero (i.e., for tensile stress paths), A(S) = f_t and the original formulation by Ortiz (1985) is retained. When only compression stresses are involved, ξ approaches infinity, and A(S) becomes equal to uniaxial compressive strength, f_c. A(S) takes on a value between f_t and f_c for mixed cracking mode.

3 Shrinkage

Shrinkage is caused by the evaporation of free water that is not needed for the hydration of cement paste. The rate and completeness of drying depends on ambient temperature, humidity, and the surface that is available for the heat flux (Troxell et al. [18]). Based on a reasonable correlation of shrinkage strains obtained experimentally Picket [19] suggested that shrinkage deformation of concrete follows approximately the laws of diffusion similar to those governing heat conduction. Hanssen and Mattock [20] followed the approach of Picket [19] and specifically investigated the shrinkage characteristics of structural members made from the same batch of concrete but with different surface areas and volumes. They concluded that members with higher surface-to-volume ratios showed higher shrinkage deformations. Shrinkage deformations result in microcracks, and in this sense, can be interpreted as initial damage to the structure.

These investigations do not address the effect of shrinkage cracking on material and structural stiffness. Material parameters for plain concrete are normally obtained from standardized cylindrical tests, an approach that may not be appropriate. For example, when Endebrock et al. [1] tested model shear walls with the geometry shown in Fig. 2, the structural stiffness was significantly lower than the stiffness obtained by using material parameters that were obtained by testing a cylinder formed from the same batch of concrete. The work of Hanssen and Mattock [20] involving volume-to-surface ratios indicates that shrinkage deformation associated with the model shear wall would be significantly larger than that of the cylinder. In this paper a method is proposed for taking into account the effect of this initial damage.

Suppose $\mathbf{F}^c$ is the initial flexibility tensor based on conventional cylinder tests. If isotropy is assumed, then

$$\mathbf{F}^c = \frac{1}{E^c}(1-\nu^c)\mathbf{I} - \frac{\nu^c}{E^c}\mathbf{i}\otimes\mathbf{i} \qquad (18)$$

in which Ec and nc denote Young’s modulus and Poisson’s ratio, respectively. To account for shrinkage cracks, let the initial flexibility tensor for the model shear wall be

$$\mathbf{F}^0 = \frac{1}{E_0}(1-\nu_0)\mathbf{I} - \frac{\nu_0}{E_0}\mathbf{i}\otimes\mathbf{i} \qquad (19)$$

where

$$\frac{1}{E_0} = \frac{\beta_1}{E^c} \quad \text{and} \quad \frac{\nu_0}{E_0} = \beta_2\frac{\nu^c}{E^c} \qquad (20)$$

In other words, $ß_1$ and $ß_2$ are dimensionless parameters designed to reflect the additional factor of flexibility introduced by shrinkage cracks. Since shrinkage cracks are primarily located near the surface, the modified flexibility tensor must be interpreted as one that represents average properties suitable for plane stress or plane strain.

4 Computational approach

A conventional stiffness approach associated with the finite element method was used to analyze the model shear wall. Rectangular elements were adopted with 2 x 2 Gaussian numerical quadrature used to obtain the element stiffness matrix. The element stiffness at the end of the previous load step was used to compute the system stiffness matrix for the current (small) load step. Strain increments were obtained and used in the constitutive equation subroutine to update the damage parameter and the element stiffness matrices. An iterative algorithm was used to ensure that the stresses satisfied the equation for the damage surface if damage was indicated. The plane stress restriction was enforced.

Although nonlinear behavior is reflected in the load-displacement curve, at no time was the load increased to the point where the stiffness matrix became singular. However, certain elements did go into the softening regime that would indicate that the corresponding regions in the wall are susceptible to the formation of macrocracks. However, this aspect was not investigated because softening is associated with localization, and a nonlocal feature must be included in the analysis to provide the proper amount of energy dissipation. These topics are well beyond the scope of the investigation.

5 Results

The shear-wall part of the model structure shown in Fig. 2 was modeled with a series of increasingly refined mesh configurations consisting of 4(2,2), 8(2,4), 16(4,4), 24(4,6), and 32(4,8) uniform elements. In each bracket, the first number denotes the number of elements used to discretize the side of the shear wall with a length of 191mm (7.5 inches). The second number represents the number of elements that discretized the side of length 457mm (18 inches). The result of a convergence study is shown in Fig. 3 for typical elastic properties.

Test results from cylindrical specimens were used to determine Young's modulus and Poisson's ratio. The parameters $ß_1$ and $ß_2$ were determined by trial and error so that the initial slope of the predicted load-deflection curve matched that of experiments on the model shear wall. For these results, $ß_2$ had no significant effect, which is not surprising since the shear modulus should be the dominant parameter for this problem. Therefore, $ß_2$=1 was used throughout and $ß_1$ suffices to reflect the damage due to shrinkage cracks. The material parameters, including $ß_1$ used for the final analysis are the following: E^0= 27000Mpa (4000 ksi), v^0= 0.2, f_c = 27Mpa (4 ksi) (uniaxial compressive strength), f_t = 3.5Mpa (0.5 ksi) (uniaxial tensile strength), α = 0.00112, and $ß_1$= 1.25. It was further assumed that all elements had the same distributed shrinkage damage for this study. If the experimental records would show otherwise, different shrinkage damage values must be assigned to different damaged elements.

Predicted and experimental load-deflection curves are shown in Fig. 4. The good match of the slope near the origin is a direct consequence of the use of the parameter, $ß_1$. The discrepancy for larger values of load may be attributed to the fact that the effect of reinforcing

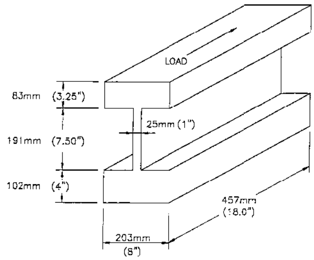

Figure 2.

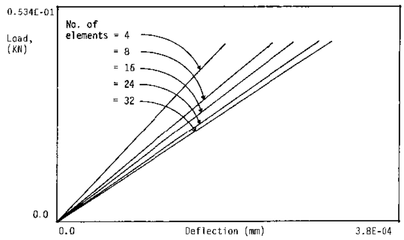

Figure 3.

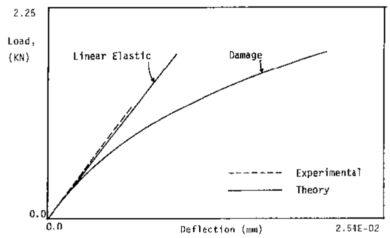

Figure 4.

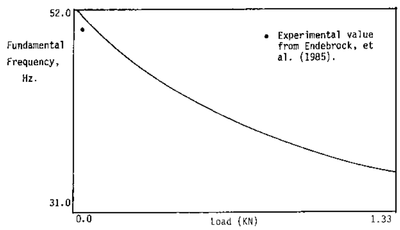

Figure 5.

bars was not included in the finite element analysis. Also, the experimental data corresponded to a very low levels of loads that the model shear walls were subjected to. It is plausible to assume that at higher stress levels larger deflection would be observed due to load induced cracks (damage) in walls.

With the system stiffness matrix obtained at various levels of load required to obtain the plot in Fig. 4 and with the development of the system mass matrix, a general eigenvalue subroutine was used to obtain the fundamental frequency. The decay in the frequency as a function of load (or damage) is shown in Fig. 5. Although only one experimental value of fundamental frequency is shown, and that frequency is based on essentially no damage, there is already a significant reduction from the natural frequency based on linear elasticity even after the reduction due to shrinkage cracks is taken into account. A plausible explanation is that the load reversals caused by the shake table have caused some debonding of the rebars and that the natural frequency actually reflects the structural property of plain concrete rather than reinforced concrete. If this hypothesis is verified with additional experiments, then the model used to predict damage illustrated in Fig. 4 might prove to be an appropriate one for predicting natural frequencies after an initial load has been applied to a reinforced concrete structure.

6 Conclusion

Damage mechanics is a relatively new field with very few demonstrated examples of applications to significant engineering problems. One example of damage involving an important engineering problem is that of the change in natural frequencies of reinforced concrete structures subjected to seismic motion with the consequence that piping structures may not be suitably designed. The result of this investigation is that a constitutive equation based on continuum damage mechanics may provide the means for evaluating these potential changes in natural frequencies. Conversely, nondestructive tests that provide natural frequencies may be used in an inverse manner to provide information on the degree of damage existing in a structure subjected to an unusual load.

This present work is considered as a feasibility study for employing an anisotropic damage model for the analysis of a structural system. To this end, a combined analytical, experimental, and numerical effort is required to further examine the merits of the proposed approach. In particular, the effects of reinforcements must be included in the analysis to realistically predict the overall response of a prototype shear wall in the case of seismic events.

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