AN EFFICIENT SIMULATED ANNEALING ALGORITHM FOR REGIONAL WASTEWATER SYSTEMS PLANNING

João A. Zeferino [1], António P. Antunes [2], and Maria C. Cunha [3]

ABSTRACT

At present, most wastewater systems are local, i.e., each city has its own system. However, in many cases, they could be better, both from the economic and the environmental viewpoints, if they were regional. The search for the best regional wastewater systems can only be effective if it is made through an optimization model. In this article, we report a study made to develop an efficient simulated annealing algorithm for solving a regional wastewater systems planning model – that is, a model aimed at determining the minimum-cost configuration for the system needed to drain the wastewater generated by the population centers located within a region, while meeting the quality standards defined for the receiving water bodies and complying with all other relevant regulatory aspects. Because of their highly non-linear nature, even moderate-size instances of a model of this type must be handled through heuristic algorithms. The simulated annealing algorithm is termed efficient because its parameters were calibrated to ensure the best possible solutions to the optimization model. The calibration was made for a sample of test problems using a particle swarm algorithm.

KEY WORDS

Wastewater system, planning model, heuristic method, simulated annealing, particle swarm.

INTRODUCTION

One of the greatest challenges the World faces today relates to the goal of giving access to drinking water and basic sanitation to a very significant part of the planet's population (UN, 2005; WHO, 2005). In order to meet this goal, numerous wastewater systems will have to be built in the near future. At present, most wastewater systems are local, i.e., each city has its own system. However, in many cases, they could be better, both from the economic and the environmental viewpoints, if they were regional.

The search for the best regional wastewater systems can only be effective if it is made through an optimization model, because the number of options available is excessively large

[1] Researcher, Civil Engineering Department, University of Coimbra, 3030 – 290 Coimbra, Portugal, Phone (+351) 239 797131, FAX (+351) 239 797142, zeferino@dec.uc.pt

[2] Professor, Civil Engineering Department, University of Coimbra, 3030 – 290 Coimbra, Portugal, Phone (+351) 239 797139, FAX (+351) 239 797142, antunes@dec.uc.pt

[3] Professor, Civil Engineering Department, University of Coimbra, 3030 – 290 Coimbra, Portugal, Phone (+351) 239 797125, FAX (+351) 239 797125, mccunha@dec.uc.pt

to enable for individual evaluation. In order to represent the problems to solve as accurately as possible, the model must incorporate discrete variables (regarding, for instance, the possible locations of treatment plants and the commercial diameters of sewers) and non-linear functions (regarding, for instance, the hydraulic behavior of wastewater systems). That is, it is necessary to resort to a discrete non-linear optimization model. Even for small-size instances, a model of this type is extremely difficult to solve. Therefore, it must be handled through heuristic algorithms. Since the 80s, numerous heuristic algorithms (e.g., genetic algorithms, tabu search, and simulated annealing) have been successfully developed to determine optimum or near-optimum solutions to discrete non-linear optimization models, (Aarts and Lenstra, 2003; Michalewicz and Fogel, 2004). In particular, simulated annealing algorithms have been successfully applied to several hydraulic systems planning models (e.g.: Cunha and Sousa, 1999; Monem and Namdarian, 2005).

In this article, we report a study made to develop an efficient simulated annealing algorithm for solving a regional wastewater systems planning model. The algorithm is termed efficient because its parameters were calibrated to ensure the best possible solutions to the planning model. The calibration was made for a sample of test problems using a particle swarm approach.

PLANNING MODEL

The study reported in this article is based on the regional wastewater systems planning model (RWSPM) presented in Sousa *et al*. (2002). This model was developed to deal with the following problem: find the minimum-cost configuration for the system needed to drain the wastewater generated by the population centers (wastewater sources) located within a region, while meeting the quality standards defined for the receiving water bodies (rivers, lakes, etc.) and complying to all other relevant regulatory aspects. The components of a wastewater system are: one or more sewer networks to connect the communities with the receiving water bodies; treatment plants to process wastewater before sending them to the receiving water bodies; and pump stations to elevate wastewater if it is unfeasible or uneconomic to drain them by gravity. The solution to the RWSPM specifies the layout of the sewer networks, the diameter of sewers, the location, type, and capacity of treatment plants, and the location and capacity of possible pump stations.

The formulation of the model is as follows:

$$\text{Min} \sum_{i=1}^{N}\sum_{j=1}^{N} C_{ij}\left(Q_{ij}, L_{ij}, E_i, E_j, x_{ij}\right) + \sum_{k=m+1}^{N} C_k\left(QT_k, y_k\right) \tag{1}$$

subject to

$$\sum_{j=1}^{N} Q_{ji} - \sum_{j=1}^{N} Q_{ij} = -QR_i, \quad \forall i = 1,\ldots.,n \tag{2}$$

$$\sum_{j=1}^{N} Q_{jl} - \sum_{j=1}^{N} Q_{lj} = 0, \quad \forall l = n+1,\ldots.,m \tag{3}$$

$$\sum_{j=1}^{N} Q_{jk} - \sum_{j=1}^{N} Q_{kj} = QT_k, \quad \forall k = m+1,....,N \tag{4}$$

$$\sum_{i=1}^{n} QR_i = \sum_{k=m+1}^{N} QT_k \tag{5}$$

$$Q\min_{ij} .x_{ij} \le Q_{ij} \le Q\max_{ij} .x_{ij}, \quad \forall i = 1,...,N; \forall j = 1,....,N \tag{6}$$

$$QT_k \le QT\max_k .y_k, \quad \forall k = m+1,...,N \tag{7}$$

$$x_{ij} \in \{0,1\}, \quad \forall i = 1,....,N; \forall j = 1,....,N \tag{8}$$

$$y_k \in \{0,1\}, \quad \forall k = m+1,....,N \tag{9}$$

$$QT_k \ge 0, \quad \forall k = m+1,...,N \tag{10}$$

$$Q_{ij} \ge 0, \quad \forall i = 1,...,N; \forall j = 1,....,N \tag{11}$$

where N: total number of nodes (wastewater sources plus possible intermediate nodes plus possible treatment plants); 1, …, n: wastewater sources; n+1, ..., m: locations for possible intermediate nodes; m+1, ..., N: locations for possible treatment plants; C_{ij}: discounted costs for installing, operating, and maintaining a sewer linking node i to node j; Q_{ij}: flow carried from node i to node j; L_{ij}: length of the sewer linking node i to node j; E_i and E_j: hydraulic heads at nodes i and j respectively; C_k: discounted costs for installing, operating and maintaining a treatment plant k; QT_k: amount of wastewater treated at treatment plant k; QR_i: amount of wastewater produced at node i; $Q\min_{ij}$ and $Q\max_{ij}$: minimum and maximum flow allowed in the sewer linking node i to node j respectively; $QT\max_k$: maximum amount of wastewater that may be treated in treatment plant k; x_{ij}: binary variable that is equal to one if there exists a sewer linking node i to node j, and equal to zero otherwise; y_k: binary variable that is equal to one if there exists a treatment plant in node k, and equal to zero otherwise.

The objective-function (1) of this discrete non-linear optimization model expresses the minimization of the total discounted costs for installing, operating, and maintaining sewer networks and treatment plants. The first term corresponds to sewer network costs, which depend on the wastewater flow (thus, on the diameter of sewers), on the length of sewers, and on the hydraulic heads at the extremities of sewers. The network may require pump stations to carry wastewater from low- to high-head points. The second term corresponds to treatment plant costs, which, for a given type of treatment plant, depend on the amount of wastewater treated there. Constraints (2), (3), and (4) are the continuity equations for three types of network nodes: wastewater sources; possible intermediate nodes; and possible treatment plants. Constraint (5) ensures that all the wastewater generated by the population centers of the region will be treated. Constraints (6) guarantee that the flow carried by sewers will be within given minimum and maximum values. These values depend both on the diameter and slope of sewers, and on flow velocity requirements. The hydraulic calculations needed to determine the diameter and slope of sewers are based on the Manning-Strickler

Equation. Constraints (7) ensure that the wastewater sent to any treatment plant will not exceed given maximum values. These values depend on the quality standards defined for the receiving water bodies. Constraints (8) and (9) are zero-one constraints. And constraints (10) and (11) are non-negativity constraints.

SOLUTION ALGORITHM: SIMULATED ANNEALING

The method designed to solve the RWSPM is based on a simulated annealing (SA) algorithm improved with a local search (LS) algorithm (Kirkpatrick *et al.*, 1983; Dowsland, 1993). The basic steps of the method are identified in Figure 1 (Left). The SA algorithm starts from some initial incumbent solution. Then, a candidate solution is selected in the neighborhood of the incumbent solution. This solution becomes the incumbent solution with probability given by the Boltzmann-Gibbs distribution; that is, $p = \min \{1, \exp(\Delta V/t)\}$, where ΔV is the difference between the value of the incumbent solution and the value of the candidate solution, and t is a parameter called temperature in a SA context. Therefore, the candidate solution becomes the incumbent solution if its value exceeds the value of the incumbent solution. Otherwise, if it does not, the probability that it becomes the incumbent solution increases as the difference of value between the solutions decreases, and, also, as the temperature decreases. This operation is repeated while decreasing the temperature in a controlled manner until the value of solutions ceases to increase. The LS algorithm starts with the best solution identified through the SA algorithm as the incumbent solution and moves into the best solution in the neighborhood of the incumbent solution if its value exceeds the value of the incumbent solution.

The three main aspects involved in the implementation of a SA algorithm are: definition of the initial incumbent solution; definition of the neighborhood of an incumbent solution; and definition of the cooling schedule (initial temperature, temperature decrease rate, and final temperature). For the RWSPM, these aspects were handled as follows. The initial incumbent solution is defined installing treatment plants at every treatment node and connecting the population centers to the closest treatment node, as shown in Figure 2 (Left). The neighborhood of an incumbent solution consists of every solution that can be reached by selecting a sewer and replacing its downstream node with one of the nodes adjacent to the upstream node, as shown in Figure 2 (Center and Right). In this figure, sewer *a* was selected and replaced by *a*', thus leading to a major change of the network (if sewer *b* was selected and replaced by *b*´ the change would have been minor). The cooling schedule was defined with four parameters, α_1, λ, γ, and σ, as proposed in Johnson *et al.*, 1991. Parameter α_1 sets the initial acceptance rate for candidate solutions with value 10-percent smaller than the value of the incumbent solution. Parameter λ sets the minimum number of candidate solutions that must be evaluated at each temperature (if after $\lambda \times S$ evaluations, where S is the number of possible sewers, the best solution value, V^*, or the average solution value, m_V, did not improve, the temperature decreases). Parameter γ sets the rate at which the temperature decreases. Finally, parameter σ sets the maximum number of temperature decreases that may occur without an improvement of the best or the average solution value. The way the parameters interact is described in Figure 1 (Right).

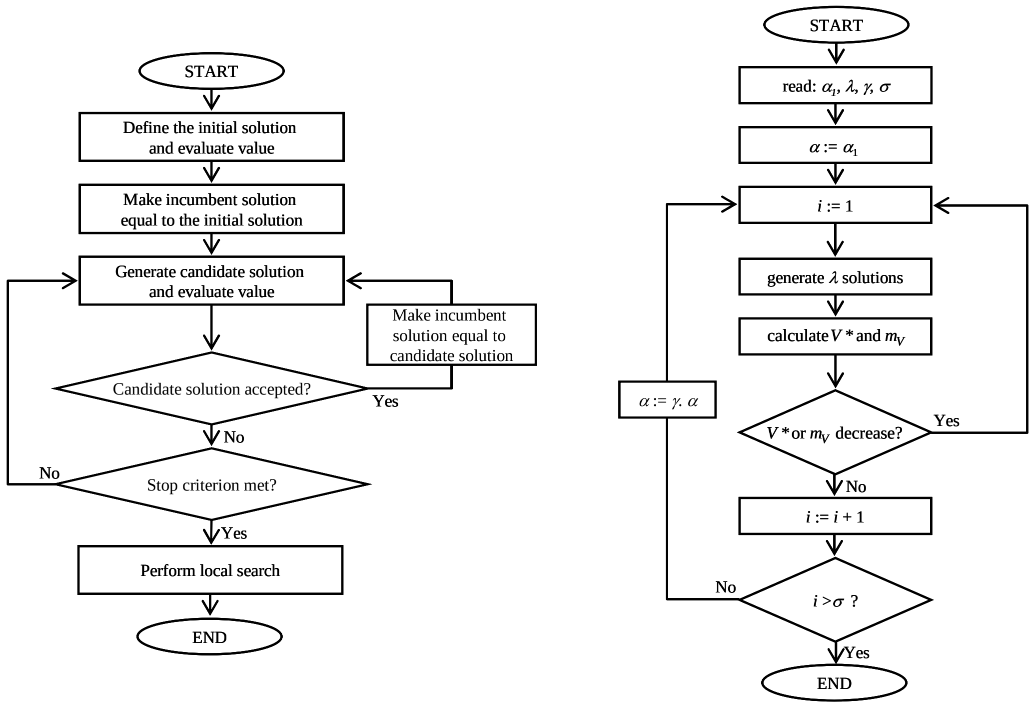

Figure 1: Annealing Algorithm - Basic Steps (Left) and Cooling Schedule (Right)

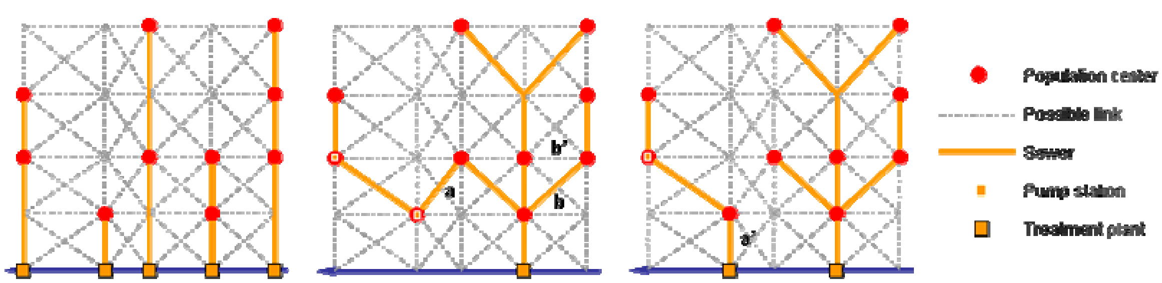

Figure 2: Initial Solution (Left), Incumbent Solution (Center) and Candidate Solution (Right)

TEST PROBLEMS

The aptitude of a SA algorithm for finding optimum or near-optimum solutions within acceptable computing effort largely depends on the way it is implemented for the particular model to solve. The implementation should be made for a sample of test problems with characteristics similar to those of real-world problems. The test problems considered for the study reported in this article were defined according to rules regarding the shape and topography of the regions, the location and size of population centers, the wastewater generation rate, the location and maximum discharge at treatment plants, and the costs of the components of the system.

Shape and Topography of the Regions

The regions have a rectangular shape, with the length of each side randomly chosen, in terms of a uniform distribution, in the interval [20, 40] km. The bottom side corresponds to a river that receives the wastewater generated by the population centers of the region. The topography of the region is based on a grid of axis spaced of a length randomly chosen in the interval [3, 6] km (this means that the size of the grid can go from 4X4 to 13X13 axis, and is, on average, 7X7 axis). The height corresponding to the nodes of the grid vary between a value of zero in the left bottom corner (river mouth) and a value randomly chosen in the interval [100, 500] m. From the mouth of the river, the height increases or decreases in both directions proportionally to a value randomly chosen in the interval [0, 6] or [0, 3] units. As a result of this, on average, the height increases 1.5 units in both directions. In order to guarantee a single value for the height in each node, a weight, *w*, randomly chosen in the interval [20, 80] % is applied to the variation of heights in the direction of the river, and a weight of (100-*w*) % is applied to the variation of heights in the orthogonal direction. The dominant orientation of the ridges is the direction that receives the larger weight. The height along the river increases proportionally to a value randomly chosen in the interval [1, 2] units. Figure 3 (Top) shows examples of the shape and topography for three test problems.

Location and Size of Population Centers

Population centers are located in a percentage of the nodes of the grid (not coincident with the river) randomly chosen in the interval [25, 75] %. The population of each center is determined in the following way: the population of the largest center is calculated by multiplying the number of centers with a value randomly chosen in the interval [5000, 15000]; the population of the second-largest center is obtained by dividing the population of the largest center by a value randomly chosen in the interval [1.5, 2.5]; the population of the third-largest center is obtained by dividing the population of the largest center by a value randomly chosen in the interval [2.5, 3.5]; and so forth. That is, the expected population distribution across centers follows a law frequently observed in real-world situations: the Zipf's law (Brakman *et al*., 2001). Figure 3 (Bottom) shows examples of the location and size of population centers for three test problems.

Wastewater Generation Rate

The wastewater generation rate was assumed to be equal for all population centers with a value of 200 l/inhabitant.

Location and Maximum Discharge at Treatment Plants

Treatment plants can be setup in any node of the river. The maximum discharge in each plant (defined to guarantee the quality standards that must be verified in the river) is obtained through the division of the total volume of wastewater generated in the population centers of the region by a value randomly chosen in the]0.0, 3.0] interval. If this value is less than 1.0, it may be enough to setup one treatment plant; if it is greater than 2.0 it will be necessary to setup at least three treatment plants.

COSTS OF THE COMPONENTS OF THE SYSTEM

The costs of the components of the system – sewer networks, treatment plants, and possible pump stations – were established on the basis of Portuguese values. The sewer network costs include construction and maintenance. The treatment plant and pump station costs include construction, electro-mechanic equipment, maintenance and operation (in particular, energy).

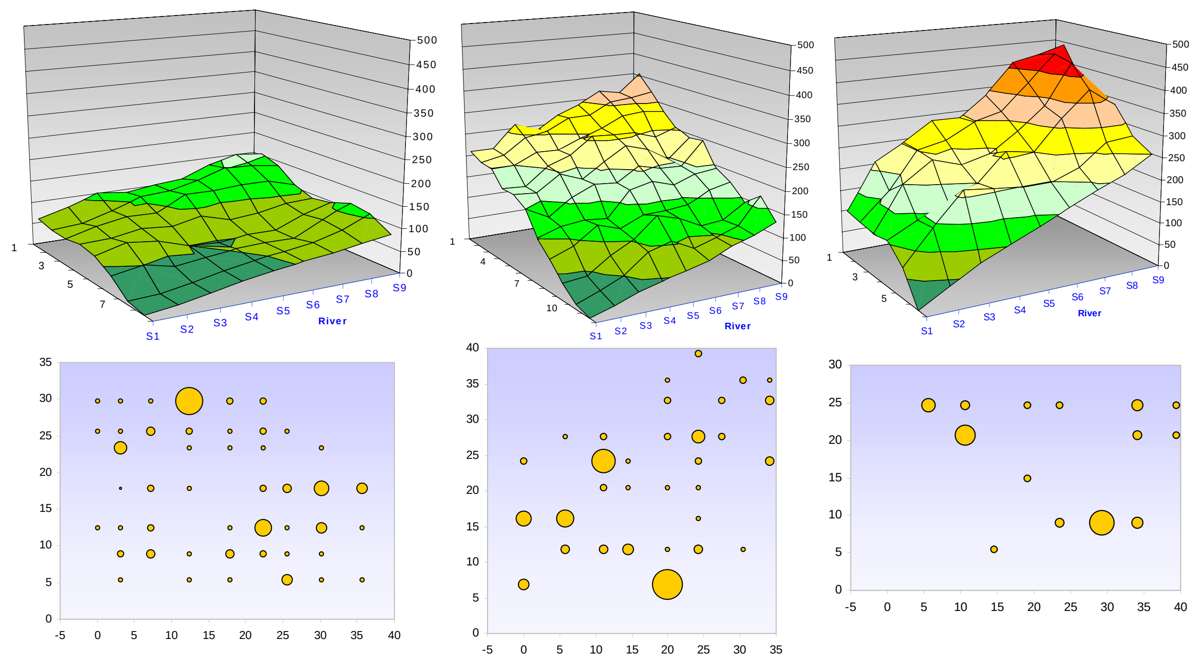

Figure 3: Shape and Topography (Top), and Location and Size of Population Centers (Bottom) for Three Test Problems

PARAMETER CALIBRATION

The procedure used to calibrate the SA parameters involved the following steps. First, we analyzed the extent to which the parameters are interrelated. Then, we developed an algorithm to determine optimum or near-optimum values for the parameters. Since some of the parameters were interrelated to a certain extent, we decided to adopt a procedure based on a Particle Swarm (PS) algorithm (Kennedy and Eberhart, 1995; Parsopoulos and Vrahatis, 2002). Finally, we determined the values for the parameters using this algorithm. These values depend on the characteristics of the problems to solve.

RELATIONSHIP BETWEEN PARAMETERS

The analysis of the relationship between the parameters was made for a sample of 20 test problems using a quadratic form multiple regression model. The quadratic form was used because it permits to identify cross-relations between the parameters. For each problem, 50 sets of parameters and 5 random seeds were used. The values of the parameters were randomly chosen within reasonable intervals. The multiple regression model was:

$$V = a_1 \times \alpha_1^{\,2} + a_2 \times \lambda^2 + a_3 \times \gamma^2 + a_4 \times \sigma^2 + \\ + a_5 \times \alpha_1 \times \lambda + a_6 \times \alpha_1 \times \gamma + a_7 \times \alpha_1 \times \sigma + a_8 \times \lambda \times \gamma + a_9 \times \lambda \times \sigma + a_{10} \times \gamma \times \sigma + \\ + a_{11} \times \alpha_1 + a_{12} \times \lambda + a_{13} \times \gamma + a_{14} \times \sigma \quad (12)$$

where V: value of the solution (cost of the wastewater system); a_1, …, a_{14}: model coefficients.

The model was always able to capture the influence of the parameters on the value of the solutions with great accuracy. Indeed, the adjusted correlation coefficient for the model was greater than 0.98 for all the 20 test problems. The *t*-tests performed on the model coefficients revealed that, with regard to the product terms, only the coefficients for $\alpha_1 \times \sigma$ and $\gamma \times \sigma$ were, in most cases, significantly different from zero for the 95% and, especially, the 99% confidence interval (Table 1). This indicates that α_1 and σ and γ and σ are the only clearly interrelated parameters.

Table 1: Relationship between Parameters

Model term	Number of times model coefficients were significantly different from zero	
	Confidence 95%	Confidence 99%
$\alpha_1 \times \lambda$	7	3
$\alpha_1 \times \gamma$	7	4
$\alpha_1 \times \sigma$	12	8
$\lambda \times \gamma$	6	4
$\lambda \times \sigma$	5	2
$\gamma \times \sigma$	13	11

CALIBRATION ALGORITHM: PARTICLE SWARM

For the calibration of the parameters, a PS algorithm was used. This type of algorithm is inspired on the way the members of a swarm synchronize their movements to achieve some objective. A PS algorithm consists of the following steps. First, a population of solutions, $\boldsymbol{S}$, is generated. Each solution is characterized by a position P in $\boldsymbol{D}$-dimensional space, with some value in terms of the objective, and a velocity, V. The velocity is the rate at which the position changes. Then, in successive iterations, each solution changes the position at a velocity that depends on its previous velocity, on the best position it has previously taken (P^*_{sd}), and on the best overall position taken by any of the solutions (P^*_{gd}). The procedure ends when, after some iterations, the change of the position taken by the solutions becomes very small (that is, the velocity becomes close to zero). The expressions used to calculate the velocity and the position of a solution $s \in \boldsymbol{S}$ in the dimension $d \in \boldsymbol{D}$, in iteration i, are:

$$V_{sd}{}^{i} = a \times V_{sd}{}^{i-1} + b \times \left(P^{*}{}_{sd} - P_{sd}{}^{i-1}\right) + b \times \left(P^{*}{}_{gd} - P_{sd}{}^{i-1}\right) \quad (13)$$

$$P_{sd}{}^{i} = P_{sd}{}^{i-1} + V_{sd}{}^{i} \quad (14)$$

where *a* and *b*: parameters.

In our implementation of the PS algorithm, each solution comprised four dimensions, the SA parameters α_1, λ, γ, and σ. The size of the population was 10. The number of iterations was 10 (for this number of iterations the velocity was already close enough to zero). The PS parameters *a* and *b* were set at 0.6 and 0.2. The initial position of the solutions over the four dimensions was randomly chosen within plausible limits for the variation of the parameters: α_1 was assumed to be in the interval [0.1, 0.5]; λ in [1, 40]; γ in [0.1, 0.9]; and σ in [1, 10]. The initial velocity of the solutions was randomly chosen within ±1/6 of the range of each parameter.

VALUE OF THE PARAMETERS

The purpose of the study reported in this paper was to determine the best possible values for the SA parameters as a function of the characteristics of the problems to solve (in particular, geographic characteristics). These values were calculated through multiple regression analysis for 20 test problems. The dependent variables considered for the analysis were the values of the SA parameters (except σ) obtained for five different seeds through the PS algorithm. The parameter σ entered as an independent variable in the explanation of the parameters with which it is correlated (that is, α_1 and γ). The number of test problems is not yet sufficient to arrive at definitive results, but will be enlarged in the future. The model used for the analysis was:

$$\xi = a_1 \times N + a_2 \times U + a_3 \times P + a_4 \times R + a_5 \times O + a_6 \times W + \left(a_7 \times \sigma\right) \quad (15)$$

where ξ: value of SA parameter; *N*: number of nodes; *U*: percentage of urban centers (in relation to the number of nodes); *P*: total population (10^3 inhabitants); *R*: land roughness (meters); *O*: ridge orientation (grades); *W*: maximum percentage of wastewater discharge in a treatment plant (in relation to total wastewater discharge); a_1, …, a_7: model coefficients.

The analysis revealed that the model globally explains the relationship very accurately. Indeed, the adjusted correlation coefficient for the three SA parameters was always larger than 0.88. The analysis also revealed that the relationship of the parameters to some variables was not significant. These variables were eliminated using (backward) stepwise regression analysis. After this was made, we arrived at the following expressions for the values of the parameters:

$$\alpha_1 = 0.00276 \times N + 0.00634 \times U - 0.00072 \times P \quad (16)$$

$$\lambda = 0.20619 \times N + 0.21581 \times O \quad (17)$$

$$\gamma = 0.00298 \times W + 0.04975 \times \sigma \quad (18)$$

According to these expressions, if $\sigma = 6$, the values for the other SA parameters to be used when solving the test problem corresponding to the region depicted on Figure 3 (Left), should be $a_1 = 0.313$, $\lambda = 29$, and $\gamma = 0.597$, since the region is characterized by $N = 72$, $U = 73\%$, $P = 488$ ($\times 10^3$), $O = 66$ grades, and $W = 100\%$.

MODEL RESULTS

The type of results obtained for the RWSPM with parameters of value given by expressions (16) to (18) are illustrated in Figure 4. The geographic characteristics of the problems solved are shown in Figure 3.

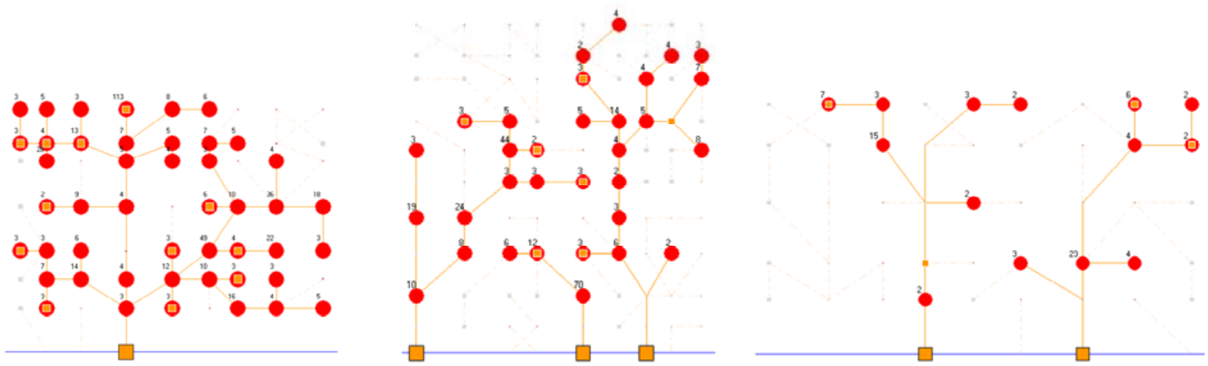

Figure 4: Solutions Obtained for Three Test Problems with the Values Proposed for the Parameters

CONCLUSION

In this article, we reported a study made to develop an efficient simulated annealing algorithm for solving a regional wastewater systems planning model. The algorithm is termed efficient because its parameters were calibrated to ensure the best possible solutions to the planning model. The calibration was made for a sample of test problems using a particle swarm approach. The values of the parameters are expressed as a function of the geographic characteristics of the regions where the planning model will be applied. The sample of test problems employed in the study was relatively small. Therefore, the expressions are not fully reliable. In the near future, a larger sample will be used to determine more reliable expressions for the value of the parameters.

ACKNOWLEDGMENTS

The authors would like to thank Fundação para a Ciência e a Tecnologia for the financial support granted to the research projects within which the study reported in this article was made – *Planeamento de Sistemas Regionais de Saneamento Básico Usando Novas Técnicas de Optimização* (PRAXIS/ECM/12266/1998) and *Planeamento e Gestão de Sistemas Regionais de Saneamento de Águas: Novos Desenvolvimentos* (POCTI/ECM/39172/ 2001).

REFERENCES

Aarts, E. and Lenstra, J. (2003). *Local Search in Combinatorial Optimisation*. Princeton University Press, Princeton, New Jersey, USA, 536 pp.

Brakman, S., Garretsen, H., and Van Marrewijk, C. (2001). *An Introduction to Geographical Economics: Trade, Location and Growth*. Cambridge University Press, Cambridge, UK, 350 pp.

Cunha, M. and Sousa, J. (1999). Water Distribution Network Design Optimization: Simulated Annealing Approach." *Journal of Water Resources Planning and Management* 125 (4) 215-221.

Dowsland, K. (1993). "Simulated Annealing." In Reeves, C. (Ed.), *Modern Heuristic Techniques for Combinatorial Problems*. John Wiley & Sons, New York (USA), 20-69.

Johnson, D., Aragon, C., Mcgeoch, L., Schevon, C. (1991). "Optimization by Simulated Annealing: An Experimental Evaluation. Part 2. Graph-coloring and Number Partitioning." *Operations Research* 39 (3) 378-406.

Kennedy, J. and Eberhart, R. (1995). "Particle Swarm Optimization." *Proceedings of the IEEE International Conference on Neural Networks*, Piscataway, New Jersey, USA, 1942-1948.

Kirkpatrick, S., Gellatt, C., Vecchi, M. (1983). "Optimization by Simulated Annealing". *Science* 220 (4598) 671-680.

Michalewicz, Z., Fogel, D. (2004). *How to Solve It: Modern Heuristics*. Springer, Berlin, Germany, 467 pp.

Monem, M. and Namdarian, R. (2005). "Application of Simulated Annealing Techniques for Optimal Water Distribution in Irrigation Canals." Irrigation and Drainage 54 (4) 365-373.

Parsopoulos, K. and Vrahatis, M. (2002). "Recent Approaches to Global Pptimization Problems through Particle Swarm Optimization." *Natural Computing* 1 (1) 235-306.

Sousa, J., Ribeiro, A., Cunha, M.C., and Antunes, A. (2002). "An Optimization Approach to Wastewater Systems Planning at Regional Level." *Journal of Hydroinformatics* 4 (2) 15-123.

UN - United Nations (2005). *UN Millenium Development Goals* (available at http://www.-un.org/millenniumgoals).

WHO - World Health Organization (2005). *Water for Life: Making it Happen*. Geneva, Switzerland, 44 pp.