

## The Computational Generation and Realization of Spatial Truss Structures

X. Z. Zhao<sup>1</sup>, R. H. Wang<sup>2</sup> and K. Shea<sup>3</sup>

<sup>1</sup>College of Civil Engineering, Tongji University, Shanghai, China, 200092; email: x.Zhao@tongji.edu.cn

<sup>2</sup>School of Civil Engineering and Architecture, Jiangsu University of Science and Technology, Zhenjiang, Jiangsu, China, 212003; email: wrhChina@163.com

<sup>3</sup>Engineering Design and Computing Laboratory CLA F, ETH Zurich, 8092 Zurich, Switzerland; email: [kshea@ethz.ch](mailto:kshea@ethz.ch)

### ABSTRACT

Computational methods have been successfully applied in structural analysis and drawings of large spatial structures. However, there are still huge challenges in performing conceptual design of structures using computers as what human brain is able to do. A computational method, named performance-based structural synthesis has been proposed and validated to generate and realize planar truss structures. However, it is insufficient for more general spatial truss structures whose geometries and load-carrying requirements are more complex compared with those of planar truss. Therefore, this computational synthesis method based on planar shape grammar was extended and presented in this paper to generate three-dimensional spatial truss structures through building 3D shape grammars and multi-objective hybridized optimization algorithm. Meanwhile techniques of load-generation were employed in the structural synthesis method to realize its engineering application. In the aspect of structural configuration, the method integrates structural topology, geometry and section size allocation; in the aspect of structural design and optimization, it takes into account the combining effects of safety, economy, construction and architectural elegance. Thus the structure generated by this synthesis method must be the best design in terms of the overall structural performance and cost compared with that obtained by traditional design method.

### INTRODUCTION

The application of computer in conceptual design of spatial truss structures remains a huge challenge, even though it has been successfully applied to drawings and structural analysis in detail design. This chasm between the two stages is partially due to the extremely complicated solution with multi-objective requirements such as

safety, economy and elegance in conceptual design, and more importantly to the lack of sufficient method to numerically describe the design process. To provide innovative alternatives for conceptual design of spatial truss structures, a computational method has been proposed based on the performance-based synthesis method STSA which has successfully generated planar truss and truss-beam (Shea, 1997; Shea and Zhao, 2004).

Formal algebra and polyhedron synthesis theory of structural morphology have been used to describe regular structural form for conventional frame structures (Nooshin and Disney, 2000), but the generation based on those approaches pays more attention to structural form rather than structural function. While the computational design synthesis (Cagan, et al., 2005) based on shape grammars (Stiny and Gips, 1972) introduces a new design idea to merge the structural form with structural function together. Based on analogy between knowledge application of structural design and the forming process of human language, a set of shape transforming rules including shape, sizing and topology rules were defined as rule syntaxes of design language, and used to realize shape composition to describe the structural form. And then the computer implementation system based on shape grammars instructs machine execute routine tasks and promote creative activities of the designer (Chau, et al., 2004), while more complex factors affecting the structural design can be considered in such synthesis system.

The performance-based structural synthesis research has exhibited that shape annealing (Cagan and Mitchell, 1993) incorporating shape grammar and simulated annealing is sufficient to generate and realize innovative planar truss (Shea, et al., 1997), dome (Shea and Cagan, 1997), and cantilever spatial truss beam (Shea and Zhao, 2004). In addition, a recently published shape annealing study on planar truss structure identified such method as an optimizing tool to design a full-scale transmission tower (Shea and Smith, 2006). These insights establish a compelling rationale to target the computational generation for general spatial truss structures.

In the aspect of optimizing algorithm, we recently proved the feasibility of targeting computational generation of spatial truss structures using improved shape annealing (ISA) (Wang, et al., 2009) that hybridizes shape annealing with fully stressed design criteria (FSD) (Patnaik and Hopkins, 1998) which is an efficient mechanical method for sizing optimization of truss structure with fixed configuration. In the aspect of topological generation of spatial truss, 3D shape grammars in addition to planar shape grammars has been explored through adding bracings, and applying load-generation technique for distributed loads including wind load and the weight of exterior-protected structures. These results indicate that the computational method for spatial truss based on the improved shape annealing, 3D shape grammars as well as load-generation technique can successfully produce innovative designs integrating structural function and configuration.

## SHAPE ANNEALING ALGORITHM FOR TRUSS GENERATION

**Introduction of STSA for truss generation (Shea and Smith, 2006).** Shape annealing was first applied to produce optimally directed shapes using the language specified by shape grammar (Cagan and Mitchell, 1993). And then that was introduced to design planar truss structure using shape grammars and optimizing algorithm of modified SA (Reddy and Cagan, 1995). Recently, it was renamed structural topology and shape annealing (STSA) (Shea and Smith, 2006), which employed more efficient annealing schedule of modified Lam-Delosm annealing schedule in addition to improved shape grammars.

Optimally directed search of the resulting design space is accomplished through the modified annealing schedule, where the accepting rate is defined as the number of accepted moves out of total moves over a fixed statistical interval. Once one shape rule has been chosen randomly based on the probability function of selecting rule, it is applied to the current design at every run. A new design will be created and be evaluated by multi-criterion objective function that consists of a multi-objective function and dynamically weighted constraint violation penalties as shown in Eq.1. Since each move to the current design may result in large disturbance in the performance and evaluation of the objective function, it may not always generate better design than the previous one. Thus whether to accept the move or not is determined in accordance with the Metropolis criteria based on the accepting probability.

$$\text{cost} = \sum_{i=1}^m (\text{objective\_weight}_i * \text{objective\_value}_i) + \sum_{j=1}^n (\text{constraint\_weight}_j * \text{constraint\_violation}_j) \quad (1)$$

Normally, the multi-objective function is the weighted summation of the mass and a penalty of constraints. Yet the cost function may exist in some different patterns altered with special problem. Penalty factor facilitates tackling variant constraints such as stress, buckling and displacement constraints. Note that all constraints are soft constraints which could be violated as the design progresses, but annealing schedule attempts to push the constraint violations to zero at the end of the iteration process while minimizing the objective cost.

**Improved STSA algorithm.** Compared with traditional design method, STSA is capable of generating innovative and efficient alternative design, but emphasizes mainly structural configuration rather than structural performance. Actually the pioneers (Reddy and Cagan, 1995) of shape annealing had pointed out such character that the grammar contributes to its inefficiency, so that one shape rule application does not always make meaningful change for the design.

To overcome the disadvantage described above, FSD is hybridized with STSA to make the heuristic searching algorithm consider structural mechanical

behavior, and established an ISA algorithm (Wang, et al., 2009). Studies show that FSD can be ignited when relatively stable topology is obtained in design, where topology rules inefficiently modify the design while sizing rules are most efficient than other rules. But the point is how to confirm that the topology is relatively stable. The efficiency of change to the design modified by each kind of rules can be utilized to confirm the stability, in accordance with the quality that is the ratio of the summation of cost change variation to the summation of the rule-applied numbers as shown in Eq. 2a. Note that the quality of each type of rules should be evaluated at the end of the outer iteration and before the starting of the next iteration.

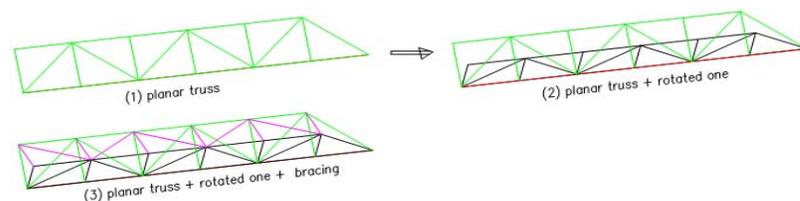
$$q_i = \frac{\sum_{j=1}^i |\Delta \text{cost}_j|}{\sum_{j=1}^i R_j}, (i = g / s / t), \quad (2a)$$

$$Q_i = \frac{q_i}{q_g + q_s + q_t}, (i = g / s / t). \quad (2b)$$

Here, the quality,  $q_i$ , indicates the impact of the  $i^{\text{th}}$  type of grammar rules applied on the tentative designs.  $R_j$  is the number of application of the  $j^{\text{th}}$  rule within  $g/s/t$  which are the abbreviations for geometry, sizing and topology rules respectively. The final quality,  $Q_i$ , represents the relative quality of a certain type of rule compared with that of other type of rules. Thus the value obtained via Eq. 2b manifests the real efficiency of one type of grammar rules. In our experience, the current candidate is relatively stable once three types of relative qualities comply with the relation as  $Q_g < Q_s < Q_t$ .

### 3D SHAPE GRAMMAR EXTENDED BY PLANAR SHAPE GRAMMAR

As far as the configuration of spatial truss beam system is concerned, the basic unit of that is not the triangle but the pyramid. The generation of spatial truss structure can refer to the transforming approach of planar truss structure, as planar grammar can be applied to describe the configuration of spatial truss beam through suitable geometrical transformation and adding bracings as shown in Figure 1 (Shea and Zhao, 2004).



**Figure 1. Synthesis of spatial truss beam with planar truss and bracing.**

Therefore, 3D shape grammars were proposed in the paper to replace pyramid by extending planar shape grammar with bracings for the description the structural form of spatial truss beam system. Then shape grammars of those can be expressed as “planar shape grammars + bracings”, as shown in Figure 2. In Figure 2, the brace-in

is the inner bracing to generate stable spatial truss beam derived from twin planar trusses via appropriate geometrical transforming, and brace-out is the outer bracing between two stable spatial truss beams.

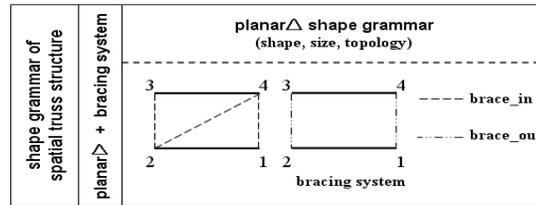


Figure 2. Shape grammars of spatial truss beam system.

**LOAD-GENERATION TECHNIQUES FOR DISTRIBUTED LOAD**

Not only wind load but also the weight of exterior-protected construction should be transferred to the load-resisting system. For dome structures, the distributed loads are convenient transform to point loads as their forms are entirely composed of triangle shapes; for spatial truss beam system produced by 3D shape grammars, those can also be tackled into point loads through appropriate geometrical segmentation as Figure 3.

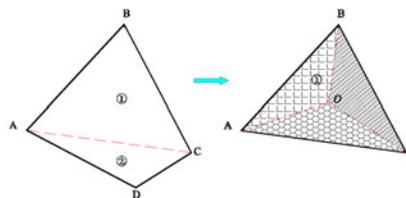


Figure 3. Segmentation of distributed load.

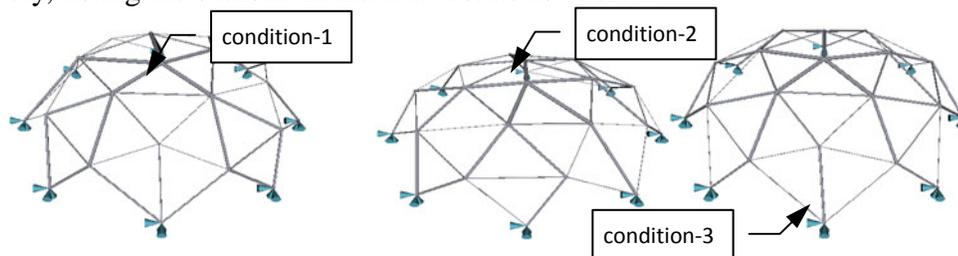
The region subjected to distributed load can be decomposed firstly into triangles as  $\triangle ABC$  and  $\triangle ADC$ . Every decomposed triangle can be re-segmented into three mini triangles as  $\triangle ABO$ ,  $\triangle BCO$  and  $\triangle CAO$ . Subsequently, the load on every mini triangle is assigned to the nearest component in the shape. Finally, the assigned loads of all components will be processed as point loads to points A, B and C respectively. Note that every point accomplishes load allocation during adding bracings between two twin planar trusses and between two spatial truss beams.

**ILLUSTRATION EXAMPLES**

**Spatial dome structure with multiple load conditions**

A spatial dome (Shea, 1997) is adopted to verify the load-generation technique for distributed load and ISA. The design space is defined as an ellipsoid with 30 m diameter span and 9.25 m height. The material is aluminum with elastic modulus of  $7.1E^6$  N/cm<sup>2</sup>, allowable tensile and compressive stresses of  $2.1E^4$  N/cm<sup>2</sup>, mass density of  $2.7E^{-3}$  kg/cm<sup>3</sup>. The design objective is to maximize the utility of envelope space while to minimize the surface area, under three load conditions as

following: 1. concentrated force of  $3.0E^5$  N at the center point of dome and gravity of members; 2. load condition 1 plus wind load of  $3.6E^{-2}$  N/cm<sup>2</sup> directly acting on members without exterior-protect structure; 3. load condition 1 plus wind load of  $3.6E^{-2}$  N/cm<sup>2</sup> indirectly acting on members with exterior-protect structure. After ten runs, the optimal configuration of three load conditions are shown as Figure 4 respectively, listing the statistic numerical results as Table 1.



**Figure 4. Optimal configuration of dome under three load conditions.**

**Table 1. Statistical results of multiple load conditions.**

load condition	mean weight	max weight	min weight	standard deviation
1	4644	6678	4644	654
2	5342	7686	4428	930
3	5529	6249	4971	404

The average weight of the ten runs is 4644 kg without wind load. Compared with that without wind load, load condition 2 with wind load directly acting on the members without envelope structure increased about 15%; while the average weight increased about 19% for load condition 3. The results demonstrate that wind load has crucial effect on dome structures.

**Spatial truss beam system.** To validate the computational synthesis method proposed based on 3D shape grammars and ISA, a spatial truss beam with a span of 54 m was explored first. And then it was used to establish initial topology for a complex spatial truss beam system, whose main size derived from the Shanghai Stadium in China with appropriate simplification through rotating and scaling transformation. The two structures use same steel material with an elastic modulus of  $2.1E^7$  N/cm<sup>2</sup>, allowable tensile and compressive stresses of  $3.1E^4$  N/cm<sup>2</sup>, and mass density of  $7.85E^{-3}$  kg/cm<sup>3</sup>. Eight concentrated loads of  $9.53E^5$  N act on eight fixed points of the upper chord. The constraints include stress and Euler buckling.

In terms of spatial truss beam, after ten runs, two representative optimal designs are selected to illustrate the capacity for conceptual design of improved synthesis system, as shown in Figure 5. The bow-shaped configurations of the two optimal designs are in agreement with the inner moment distribution.

The spatial truss beam system consisting of 32 spatial truss beams with

bracing system has symmetrical configuration, and this character facilitates the generation of such structure. The generation of such complex system undergoes two steps, one is that a pair of twin planar trusses construct the first basic spatial truss beam through geometrically rotating and bracing brace-in, the other is that a series of spatial truss beams constitute the spatial truss beam system through scaling the basic truss beam in accordance with their locations in design space and bracing brace-out between two spatial truss beams, as shown in Figure 6.

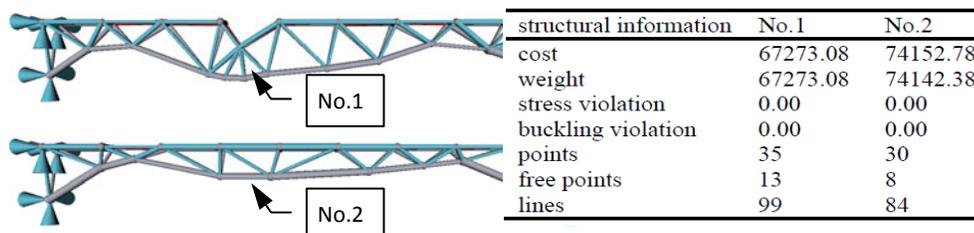


Figure 5. Generation of optimal spatial truss beam.

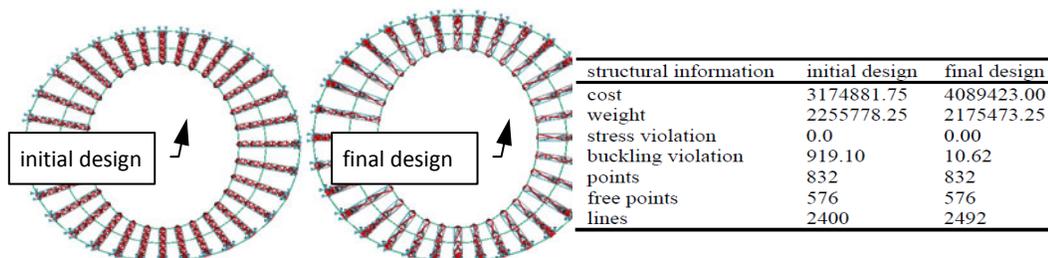


Figure 6. Generation of spatial truss beam system.

Here, only one annealing run was implemented, but that is enough to achieve the primary purpose of this paper, which is to explore a feasible method to generate computationally the general spatial truss structures. Although there is still constraints violation of Euler buckling due to the fixed number of moves in annealing schedule (Reddy and Cagan, 1995), the design has been obviously improved compared with initial design. Our experience finds that the final design can be further refined if a larger number of moves are utilized in the annealing schedule.

**CONCLUSION**

With the improvement in optimizing algorithm, shape grammar and load-generation technique, the improved computational synthesis method proposed in the paper accomplishes two complicated design tasks, structural form and function, in conceptual design for spatial truss structure. The form of spatial truss structures derived is more regular and meanwhile more rational for carrying loads. Thus the structure generated by this synthesis method must be the best design in terms of the overall structural performance and cost compared with that obtained by traditional design method.

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