Finite Element Analyses of Double-wall Sandwich Structures with Viscoelastic Core

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ABSTRACT

This paper presents a reduced order finite element model for sound transmission analysis through a double-wall sandwich structures with viscoelastic core inserted in an infinite baffle. The proposed model is derived from a multi-field variational principle involving structural displacement and acoustic pressure inside the fluid cavity. To solve the vibro-acoustic problem, the plate displacements are expanded as a modal summation of the plate's eigenfunctions in vacuo. Similarly, the cavity pressure is expanded as a summation over the modes of the cavity with rigid boundaries. Then, an appropriate reduced-order model is introduced. The structure is excited by a plane wave at the source side. An example of the normal sound transmission loss of a double glazed window with laminated glass is shown. This example illustrates the accuracy and the versatility of the proposed reduced order model, especially in terms of prediction of sound transmission.

INTRODUCTION

Double-wall panels are widely used in engineering applications in order to reduce structural vibrations and interior noise due to their superiority over single-leaf structures. Typical examples include double glazed windows, fuselage of airplanes, etc… By introducing a thin viscoelastic interlayer within the panels, a better acoustic insulation is obtained (Basten 2001). In fact, sandwich structures with viscoelastic layer are commonly used in many systems for vibration damping and noise control especially at the medium and high frequency ranges. In such structures, the main energy loss mechanism is due to the transverse shear of the viscoelastic core.

This paper describes a reduced order finite element models for the sound transmission analysis through a double sandwich panels with viscoelastic core inserted in an infinite baffle. The structure is excited by a plane wave at the source side and fluid loading is neglected. The proposed FE model is derived from a variational principle involving structural displacement and acoustic pressure in the fluid cavity. To solve the vibro-acoustic problem, the direct solution can be considered only for small model sizes. This has severe limitations in attaining adequate accuracy and wider frequency ranges of interest. Thus, a reduced order-model is then proposed to solve the problem at a lower cost. The presented methodology, based on a normal mode expansion, requires the computation of the uncoupled structural and acoustic modes. The uncoupled structural modes are the real and undamped modes of the sandwich panels without fluid pressure loading at fluid-structure interface, whereas the uncoupled acoustic modes are the cavity modes with rigid wall boundary conditions at the fluid-structure interface. Moreover, the effects of the higher modes of each subsystem can be taken into account through an appropriate so-called “static correction”. A deep study of truncation effects, acceleration of convergence and error estimation, using for example various static corrections, is presently under investigation but outside the scope of this paper. Example of the normal sound transmission loss of a double glazed window with laminated glass is shown in order to illustrate the accuracy and the versatility of the proposed reduced order model.

FINITE ELEMENT FORMULATION OF THE COUPLED PROBLEM

Local equations

Consider a double-wall structure filled with an acoustic fluid shown in Figure 1. Each wall occupies a domain Ω_{Si}, $i \in \{1, 2\}$ such that $\Omega_S = (\Omega_{S1}, \Omega_{S2})$ is a partition of the whole structure domain. A prescribed force density $\mathbf{F}^d$ is applied to the external boundary Γ_t of Ω_S and a prescribed displacement $\mathbf{u}^d$ is applied on a part Γ_u of Ω_S. The acoustic enclosure is filled with a compressible and inviscid fluid occupying the domain Ω_F. The cavity walls are rigid except those in contact with the flexible wall structures noted Σ.

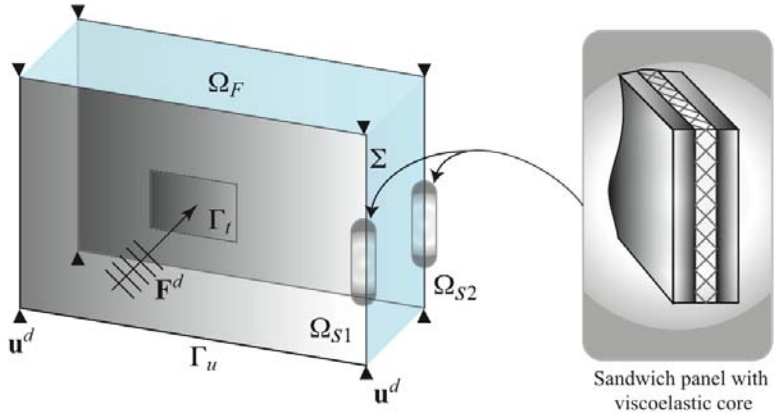

Figure. 1 Double sandwich wall structure.

The harmonic local equations of this structural-acoustic coupled problem can be written in terms of structure displacement $\mathbf{u}$ and fluid pressure field p (Morand et al. 1995, Larbi et al. 2007)

$$\text{div}\ \sigma(\mathbf{u}) + \rho_S \omega^2 \mathbf{u} = \mathbf{0} \text{ in } \Omega_S$$

$$\sigma(\mathbf{u})\mathbf{n}_S = \mathbf{F}^d \text{ on } \Gamma_t$$

$$\sigma(\mathbf{u})\mathbf{n}_S = p\mathbf{n} \text{ on } \Gamma_u$$

$$\Delta p + \frac{\omega^2}{c_F^2} p = 0 \text{ in } \Omega_F$$

$$\nabla p \cdot \mathbf{n} = \rho_F \omega^2 \mathbf{u} \cdot \mathbf{n} \text{ on } \Sigma$$

where ω is the angular frequency, $\mathbf{n}_S$ and $\mathbf{n}$ are the external unit normal to Ω_S and Ω_F; ρ_S and ρ_F are the structure and fluid mass densities; c_F is the speed of sound in the fluid; and σ is the structure stress tensor.

Constitutive relation for viscoelastic core. In order to provide better acoustic insulation and vibration attenuation, damped sandwich panels with a thin layer of viscoelastic core are used in this study (Figure 1). In fact, when subjected to mechanical vibrations, the viscoelastic layer absorbs part of the vibratory energy in the form of heat. Another part of this energy is dissipated in the constrained core due to the shear motion. The constitutive relation for a viscoelastic material subjected to a sinusoidal strain is written in the following form:

$$\sigma = \mathbf{C}^*(\omega)\varepsilon$$

where ε denote the strain tensor and $\mathbf{C}^*(\omega)$ termed the complex moduli tensor, is generally complex and frequency dependent ($*$ denotes complex quantities). It can be written as:

$$\mathbf{C}^*(\omega) = \mathbf{C}'(\omega) + i\mathbf{C}''(\omega)$$

where $i = \sqrt{-1}$.

Furthermore, for simplicity, a linear, homogeneous and viscoelastic core will be used in this work. In the isotropic case, the viscoelastic material is defined by a complex and frequency dependence shear modulus in the form:

$$G^*(\omega) = G'(\omega) + iG''(\omega)$$

where $G'(\omega)$ is known as shear storage modulus, as it is related to storing energy in the volume and $G''(\omega)$ is the shear loss modulus, which represents the energy dissipation effects. The loss factor of the viscoelastic is defined as

$$\eta(\omega) = \frac{G'(\omega)}{G''(\omega)}$$

and the Poisson's ratio ν is considered real and frequency independent. With these assumptions, the stress tensor of the sandwich structure is complex and frequency dependent.

Finite element system. After applying variational formulation to the harmonic local equations of this structural-acoustic coupled problem given in section 2.1, discretization by finite element method and using the constitutive relation for the viscoelastic core, we find the following matrix equation:

$$\left[\begin{pmatrix} \mathbf{K}_u^*(\omega) & -\mathbf{C}_{up} \\ \mathbf{0} & \mathbf{K}_p \end{pmatrix} - \omega^2 \begin{pmatrix} \mathbf{M}_u & \mathbf{0} \\ \mathbf{C}_{up}^T & \mathbf{M}_p \end{pmatrix}\right]\begin{pmatrix} \mathbf{U} \\ \mathbf{P} \end{pmatrix} = \begin{pmatrix} \mathbf{F} \\ \mathbf{0} \end{pmatrix} \qquad (1)$$

where $\mathbf{U}$ and $\mathbf{P}$ are the vectors of nodal values of $\mathbf{u}$ and p respectively, $\mathbf{M}_p$ and $\mathbf{K}_p$ are the mass and stiffness matrices of the fluid, $\mathbf{M}_u$ is the mass matrix of the structure, $\mathbf{K}_u^*(\omega)$ is the complex and also frequency dependent stiffness matrix of the structure, and $\mathbf{F}$ is the vector of nodal forces.

REDUCED ORDER MODEL

To solve the vibro-acoustic problem (Equation 1), the direct solution can be considered only for small model sizes. This has severe limitations in attaining adequate accuracy and wider frequency ranges of interest. A reduced order-model is proposed in this section to solve the problem at a lower cost. The proposed methodology, based on a normal mode expansion, requires the computation of the uncoupled structural and acoustic modes. The uncoupled structural modes are the real and undamped modes of the sandwich panels without fluid pressure loading at fluid-structure interface given by:

$$[\mathbf{K}_{u0} - \omega_{si}^2 \mathbf{M}_u]\mathbf{Y}_{si} = \mathbf{0}$$

where $\mathbf{K}_{u0}$ is the real and frequency-independent stiffness matrix calculated with a constant shear module's of the viscoelastic core such that $\mathbf{K}_u^*(\omega) = \mathbf{K}_{u0} + \delta\mathbf{K}_u^*(\omega)$, $(\omega_{si}, \mathbf{Y}_{si})$ are the natural frequency and eigenvector for the *i*-th structural mode. The uncoupled acoustic modes are the cavity modes with rigid wall boundary conditions at the fluid-structure interface given by:

$$[\mathbf{K}_p - \omega_{fi}^2 \mathbf{M}_p]\mathbf{Y}_{fi} = \mathbf{0}$$

where $(\omega_{fi}, \mathbf{Y}_{fi})$ are the natural frequency and eigenvector for the *i*-th acoustic mode.

The vibro-acoustic coupled problem given by equation (1) can be reduced by projecting the mechanical displacement into the first N_s structure eigenmodes $\mathbf{Y}_{si}$ and the acoustic pressure into the first N_f acoustic Eigenmodes $\mathbf{Y}_{fi}$, such as:

$$\mathbf{U} = \sum_{i=1}^{N_s} \mathbf{Y}_{si} q_{si}(\omega) \text{ and } \mathbf{P} = \sum_{i=1}^{N_f} \mathbf{Y}_{fi} q_{fi}(\omega) \qquad (2)$$

As a result, the reduced problem consists in solving the following system:

$$\begin{aligned} &-\omega^2 q_{si} + 2i\omega\omega_{si}\xi_{si} q_{si} + \omega_{si}^2 q_{si} + \sum_{k=1}^{N_s} \gamma_{ik}^*(\omega) q_{si} - \sum_{j=1}^{N_f} \beta_{ij} q_{fj} = F_i \\ &-\omega^2 q_{fi} + 2i\omega\omega_{fi}\xi_{fi} q_{fi} + \omega_{fi}^2 q_{fi} - \omega^2 \sum_{j=1}^{N_s} \beta_{ij} q_{sj} = 0 \end{aligned} \qquad (3)$$

where $F_i = \mathbf{Y}_{si}^T \mathbf{F}$ is the mechanical excitation of the *i*-th mode; $\beta_{ij} = \mathbf{Y}_{si}^T \mathbf{C}_{up} \mathbf{Y}_{fj}$ is the fluid-structure modal coupling coefficient; ξ_{si} and ξ_{fi} are the introduced modal damping coefficients for structure and fluid respectively, $\gamma_{ik}^*(\omega) = \mathbf{Y}_{si}^T \delta\mathbf{K}_u^*(\omega)\mathbf{Y}_{sk}$ the reduced residual stiffness complex coefficient. At each frequency step, the reduced system (3) is solved by updating $\gamma_{ik}^*(\omega)$. After determining the complex amplitude vectors q_{si} and q_{fi}, the displacement and pressure fields are reconstructed using the modal expansion (Equation (2)).

NUMERICAL EXAMPLE

The proposed reduced order finite-element formulation is applied now to calculate the transmission loss factor of a double laminated glazing window. The system consists of two identical clamped laminated panels of glass separated by an air cavity of 12 mm thickness and inserted in an infinite baffle. Each laminated glass is composed of two glass plates (0.35 mm of length and 0.22 mm of width) bonded together by a Polyvinyl Butyral (PVB) interlayer. The thickness of outer and inner glass ply is 3 mm and that of the PVB interlayer is 1.14 mm. The glass ply is modeled as linear elastic material (density 2500 kg/m^3, Young's modulus 72 GPa, and Poisson ratio 0.22). The material properties of the PVB are both thermal and frequency dependent. From dynamic and thermal tests, Havrillak and Negami have found an empirical law describing this dependence. The resulting complex frequency dependent shear modulus of the PVB is given at 20°C as (Havriliak et al. 1966, Koutsawa et al. 2012):

$$G^*(\omega) = G_\infty + (G_0 - G_\infty)[1 + (i\omega\tau_0)^{1-\alpha_0}]^{-\beta_0}$$

where G_∞=0.235 GPa, G_0=0.479 Mpa, α_0=0.46, β_0=0.1946, τ_0=0.3979. The Poisson ratio of the PVB is 0.4 and density is 999 kg/m^3.

Concerning the finite element discretization, we have used, for the structural part, 10×10 rectangular sandwich plate elements for each panel. The acoustic cavity is discretized using 10×10×5 hexahedric elements. The structural and acoustic meshes are compatible at the interface. For more details about these elements and the fluid-structure coupling element, we refer the reader to (Larbi et al. 2012, Larbi 2013)

When the excitation (a normal incidence plane wave) is applied to the first plate, the second one vibrates and radiates sound caused by the coupling of air and plate 1. The normal incidence sound transmission loss is then computed using the Rayleigh's integral method (Fahy 1985, Larbi 2013) which needs the finite element solution of surface velocities of plate 2.

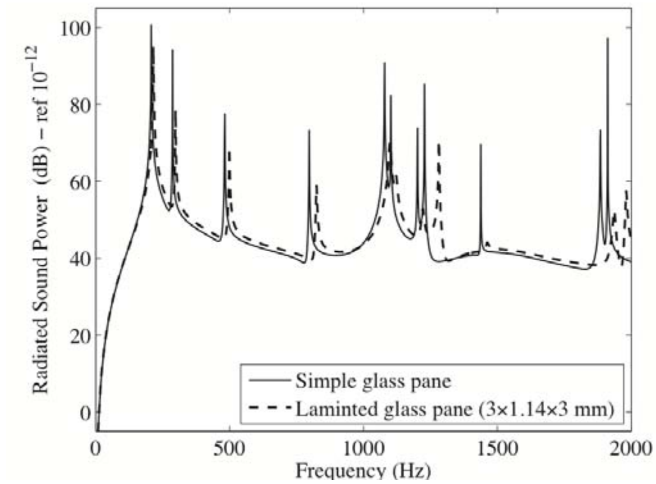

Figure. 2 Comparison of radiated sound power from a simple glass pane and a laminated glass with the same mass.

A comparison between a simple glass and a laminated glass with PVB interlayer with an equivalent surface mass is shown in Figure (2). Calculation was limited to 2000 Hz maximum. This comparison shows that laminated glass has a much lower acoustic radiation compared to conventional glass at resonance frequencies du to the effect of the viscoelastic layer. The reduction of sound radiation power is around 10 dB in lower frequencies and around 20 dB in higher frequencies. In fact, at low frequencies, the viscoelastic material is soft and the damping is small. At higher frequencies, the stiffness decreases rapidly and the damping is highest. Moreover, flexural vibrations causes shear strain in the viscoelastic core which dissipates energy and reduces vibration and noise radiation. Note that the thickness of the viscoelastic layer has a significant influence in terms of attenuation.

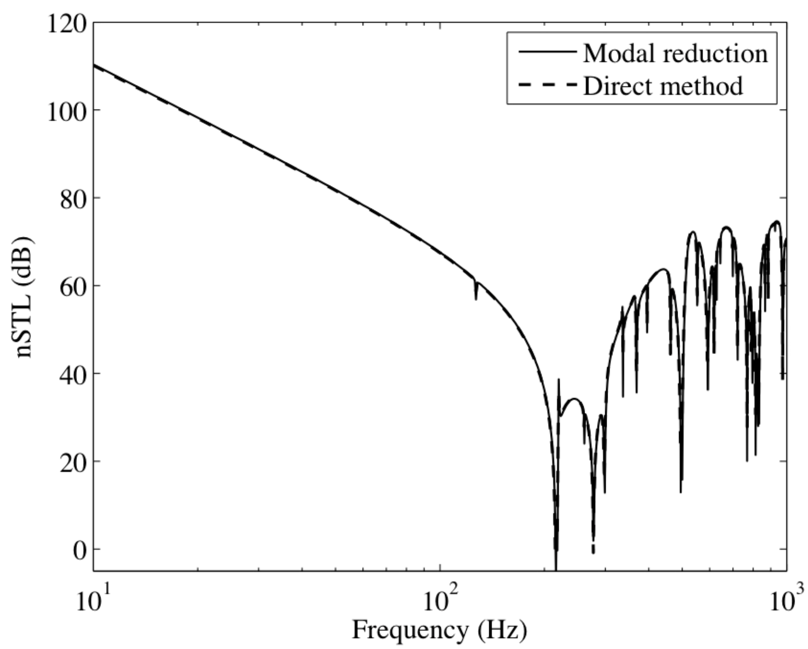

Figure. 3 nSTL through an air-filled double panel: comparison between the modal reduction approach and the direct nodal method.

Figure 3 shows a comparison between the normal incidence sound transmission (nSTL) of the coupled problem, obtained with the proposed modal reduction approach defined by Equations (3) with a truncation on the first twenty structural modes (Ns = 20) and first twenty acoustic modes (Nf = 20) and the direct nodal method (Equation (1)) where the displacement and pressure vectors are calculated for each frequency step. As can be seen, a very good agreement between the two methods is proved. In this respect, it should be noted that the resulting reduction of the model size and the computational effort using the reduced order method are very significant compared to those of the direct approach. For this example, the comparisons of computational times (using an implementation of the two methods in the Matlab software) showed that the reduced order model is much more faster than the direct technique (the CPU time ratio is about 10).

CONCLUSIONS

In this paper, a finite element formulation for sound transmission through double-wall sandwich structures with viscoelastic core is presented. A reduced-order model, based on a normal mode expansion, is then developed. The proposed methodology requires the computation of the eigenmodes of the structure *in vacuo*, and the rigid acoustic cavity. Despite its reduced size, this model is proved to be very efficient for simulations of steady-state analyses of structural-acoustic coupled systems. The Rayleigh integral method is used in order to estimate the sound transmission loss factor of the system. Examples are presented in order to validate and illustrate the efficiency of the proposed finite element formulation. Further investigations will concern introduction of (i) passive dissipation in the fluid and active one in the structure using piezoelectric materials and (ii) quasi-static corrections in the modal bases in order to accelerate the convergence.

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