

## Resiliency of Intelligent Transportation Systems to Critical Disruptions: An Eigenvalue-Based Viewpoint

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### ABSTRACT

Identification of important nodes/links is critical for understanding the resiliency of transportation networks in response to disruptions. Envisioning an intelligent transportation system requires effective monitoring of different components of a transportation network, such as nodes and links. For example, identification of critical nodes/links for placing different monitoring devices (e.g., cameras) is an important task of an intelligent transportation system. This paper is focused on identifying important nodes of a given transportation network based on a novel approach that minimizes the largest eigenvalue of an adjacency matrix of a transportation network. We tested the proposed approach on the Guam road network. Our results suggested that if capacities of 15% of the critical nodes are reduced by 50%, the connectivity of the entire network drops by 50%, which is significant performance degradation. The proposed approach is more effective than existing popular metrics (e.g., betweenness) in identifying a subset of critical nodes/links.

### INTRODUCTION

In modern societies, resiliency of networked infrastructures to large-scale collapse is critical (Albert, et al., 2000, Duenas-Osorio and Vemuru, 2009, Jenelius and Mattsson, 2012). Small local failures typically propagate through a network and lead to large-scale breakdowns, which are called macroscopic avalanches (Chen, et al., 2002). Particularly, in a road network, any disruption that leads to failure of nodes/links in a network may significantly impact the network performance (Zhu, et al., 2010). For instance, it might either cause delay in receiving medical care and other critical services, or impact daily activities of individuals. The cause of disruptions can originate either from internal causes (e.g., car crashes, or bridge collapses) leading to blockage of a single link, or from external sources (e.g., floods, landslides, snowfall, storms, earthquakes or other natural hazards) leading to partial/complete blockage of multiple links at a time. Thus, resiliency assessment of road networks is a necessary task.

Current research studies have focused on the impacts of failure of single nodes and ranked them based on their impacts. In the case of multiple link failure, researchers considered the top-ranked subset of individual links as the most important subset of links (Jenelius and Mattsson, 2012). In reality, the disruptions may

simultaneously impact multiple nodes/links spatially extended across the network. Thus, the question of interest may be to find the subset of critical nodes/links that is the smallest subset of nodes/links whose simultaneous disruption minimizes network connectivity. In this manuscript, we develop a method to identify a subset of critical nodes. To do so, we assess a macroscopic measure of network connectivity based on eigenvalues of adjacency matrix of a network. We show that such a macroscopic measure dominates other existing criteria (e.g., degree or betweenness) in identifying a subset of critical nodes.

## BACKGROUND RESEARCH

Vulnerability assessment of infrastructures was the subject of interest to several researchers. For instance, economic consequences of earthquakes in disrupting road networks have been studied using integrated transport network and multiregional trade models (Tatano and Tsuchiya, 2008). In addition, the researchers proposed different metrics and models to evaluate the impacts of disruptions on network performance (Jenelius, et al., 2006, Knoop, et al., 2008, Matisziw and Murray, 2009, Sohn, 2006). In this section, we summarize the background into two categories: (1) disruption characteristics and (2) existing importance criteria.

**Disruption characteristics.** Disruption characteristics may be listed as: (a) cause of disruption, (b) type of disruption (either single-link or multiple-links blockage), and (c) severity of disruption (either partial or complete reduction of links' capacity). Each of these characteristics is briefly discussed next.

*(a) Cause of disruption:* Different events such as car crashes to heavy snowfalls, cause degradations in the performance of road networks. The causes of disruptions can be grouped into two different categories (1) internal and (2) external causes (Jenelius and Mattsson, 2012). The internal causes include: (a) car crashes, (b) random failures such as bridge collapse, and (c) incidents due to road works, such as maintenance. The external causes include: (i) natural events, such as floods, heavy snowfalls, storms, hurricanes, and earthquakes, or (ii) anthropogenic events, such as targeted terrorist attack (Sohn, 2006).

*(b) Type of disruption:* The disruptions due to internal causes typically lead to capacity reduction (or blockage) of a single link at a time (Jenelius and Mattsson, 2012, Jenelius, et al., 2006, Knoop, et al., 2008, Sohn, 2006, Taylor, et al., 2006). For example, Knoop et al. (2008) studied impacts of an accident over a single link on overall network performance. In contrast, the disruptions caused by nature may extend across large areas and lead to capacity reduction (or blockage) of multiple links (Jenelius and Mattsson, 2012). For instance, some natural hazards such as flood, avalanches, and heavy snowfall cause joint failure of multiple links of a network (Erath, et al., 2009).

*(c) Severity of disruption:* While some common everyday disrupting events, such as partial flooding, visibility reduction and minor accidents, may cause partial reduction of links' capacity, catastrophic events such as earthquake, collapse of bridges and major accidents may cause the complete capacity reduction of affected links (Sullivan, et al., 2010). Real examples of complete blockage of links include events such as the 1995 earthquake in Kobe, Japan, the attacks on the World Trade

Center in New York City in 2001, and the I-35 bridge collapse in Minneapolis, MN in 2007 (Sullivan, et al., 2010).

**Importance criteria.** Historically, the nodes/links with largest traffic volume were considered as important nodes/links of a network. Two traditional measures of importance are: (1) the average annual daily traffic (AADT) collected from traffic count and (2) the Volume-to-Capacity (V/C) (Fhwa, 2008). The disadvantage of these measures is that they are localized measures which do not consider network-wide impacts of failure. Sullivan et al. (2010) discussed that the critical links may not necessarily be the links with highest traffic volume, but may be the links with relatively high volume and few alternate routes.

Another group of importance criteria are the measures that were defined based on the topology of road networks, such as degree and betweenness (Dall'asta, et al., 2006, Duenas-Osorio and Vemuru, 2009, Malik, et al., 1989). Another example is gamma index which is a measure of connectivity that quantifies the relation between actual number of links and the maximum number of possible links in a network (Sullivan, et al., 2010). Moreover, Knoop et al. (2008) discussed that it is important to consider spillback to find critical nodes/links (Knoop, et al., 2008). This means that the importance of nodes/links depends on both topology and traffic flow. Researchers introduced importance measures based on topology and total increase in travel time or travel distance for all drivers (Jenelius and Mattsson, 2012, Jenelius, et al., 2006, Knoop, et al., 2008, Sohn, 2006). For example, they used Network Robustness Index (NRI) which captures the delays, including waiting times and increases in actual travel times caused by disruption (Jenelius and Mattsson, 2012).

The main limitations of existing works in identifying multiple critical nodes/links are twofold. First, they did not consider simultaneous disruption of multiple nodes/links. Instead, they ranked nodes/links based on their defined importance criteria and selected the top-ranked nodes/links as critical ones. Second, they did not consider network-wide impacts of disruption, i.e., impacts of disruption on overall network connectivity. Disruption of the top-ranked links might not necessarily cause minimal connectivity.

## RESEARCH APPROACH

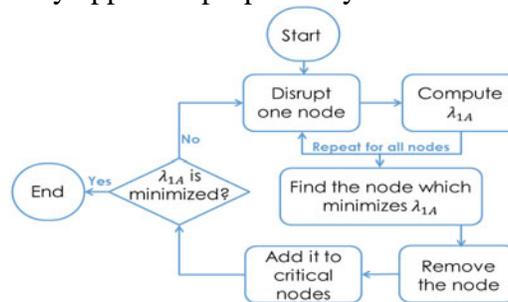
In this section, we propose an approach which enables transportation modeler to identify the subset of critical nodes/links, i.e., the nodes/links whose simultaneous disruption minimizes network connectivity. Instead of focusing on the top-ranked nodes/links, we will implement a network-wide criterion to find the critical nodes/links.

In an epidemiological study, Prakash et al. (2013) showed that the single best measure of connectivity is the largest eigenvalue of the adjacency matrix of a network (Prakash, et al., 2013). Therefore, we define criticality as the drop of the largest eigenvalue of a weighted adjacency matrix. In the weighted adjacency matrix ( $A'$ ), the weights are the capacities of the road segments in an event of disruption. Suppose the vector which gives us the distribution of  $k$  disrupted nodes be  $\vec{d} = \{d_1, d_2, \dots, d_k\}$ , where the  $d_i$  is the severity of disruption at the node  $i$ . If the matrix

$F = \text{diag}(\vec{d})$ , then the weighted matrix after disruption ( $A'$ ) is  $A' = A \times F$ , where  $A$  is the weighted adjacency matrix of the network before disruption. To find critical nodes, we identify the critical nodes whose disruption minimizes network connectivity. In summary, this problem can be reformulated as:

*Given a specific severity of disruption, find a subset of critical nodes whose disruption minimizes the largest eigenvalue of the resultant weighted adjacency matrix ( $\lambda_{A'}$ ), i.e.,  $\{d_1, d_2, \dots, d_k\} = \text{argmin}_{\vec{d}} \lambda_{A'}$*

Prakash et al. (2013) proved that the minimization of the largest eigenvalue problem is an NP-complete problem. They showed that a heuristic based on removing nodes with highest degree is not effective in terms of minimizing the exact connectivity metric,  $\lambda_{A'}$ . This heuristic ranks the nodes based on their degrees and removes the top-ranked nodes one-by-one. They concluded that the heuristic should directly attempt to minimize the largest eigenvalue  $\lambda_{A'}$ . Thus we employ an exhaustive method which is the greedy approach proposed by Prakash et al. (2013) (Figure 1).



**Figure 1. Process of Finding Subset of Critical Nodes**

We employ a validation strategy based on simulation results. Once we identify the critical nodes/links of a given road network, we reduce the capacities of critical nodes/links based on different methods in separate settings, and simulate traffic conditions using a traffic simulation model (e.g., DynusT). By investigating the change in network performance (e.g., traffic volume), we will be able to ascertain if the proposed approach perform better than the existing criteria (e.g., degree and betweenness).

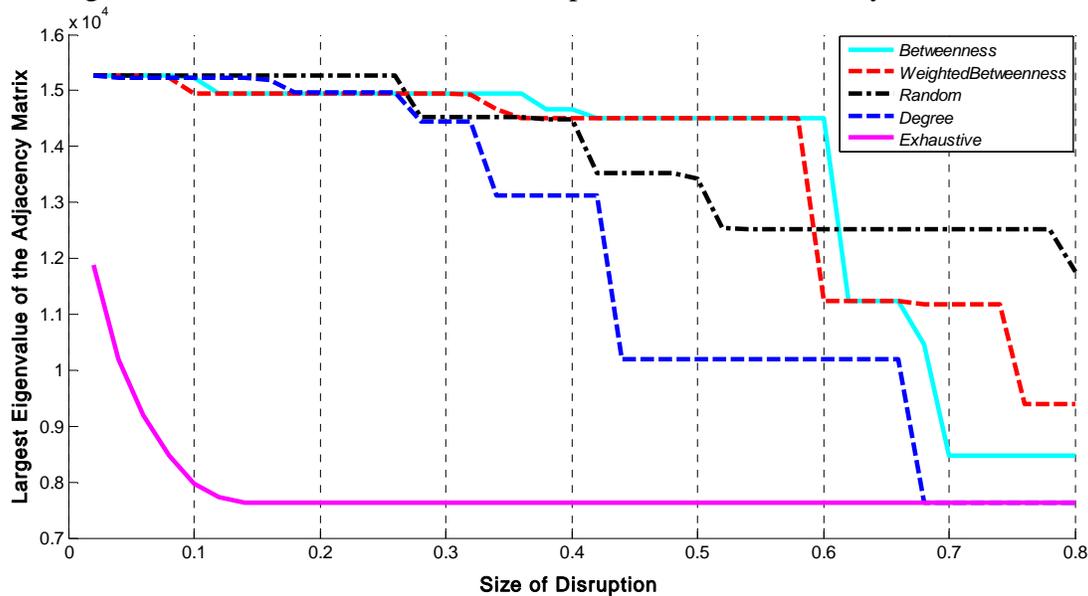
## RESULTS

In this section, we present preliminary results of using the proposed approach to find critical nodes of a given regional road network from an Island of Guam which has 539 and 1183 nodes and links. To compare with the previous works, we identified subsets of critical nodes based on various criteria, such as: (a) highest betweenness, (b) highest weighted betweenness in which the weights are the traffic flows, (c) highest degree, (d) the proposed method (Exhaustive method). Additionally, for the comparison purpose, we performed random disruption in which some randomly selected nodes were disrupted.

The algorithm was developed in MATLAB using two different packages. The degree and betweenness of the nodes were calculated using the “Complex Network

*Package*” in MATLAB (Muchnik, et al., 2007). The weighted betweenness was calculated using “*met Package*” in R (Opsahl, et al., 2010).

Figure 2 shows the largest eigenvalue of the adjacency matrix versus the size of disruption for different methods. The *size* refers to the portion of the nodes that will be impacted. For example, when 100 nodes out of 1000 links are disrupted, the size of disruption is 10%. In this research, we focused on an example of partial blockage of links. We assumed that links’ capacities were reduced by 50%.



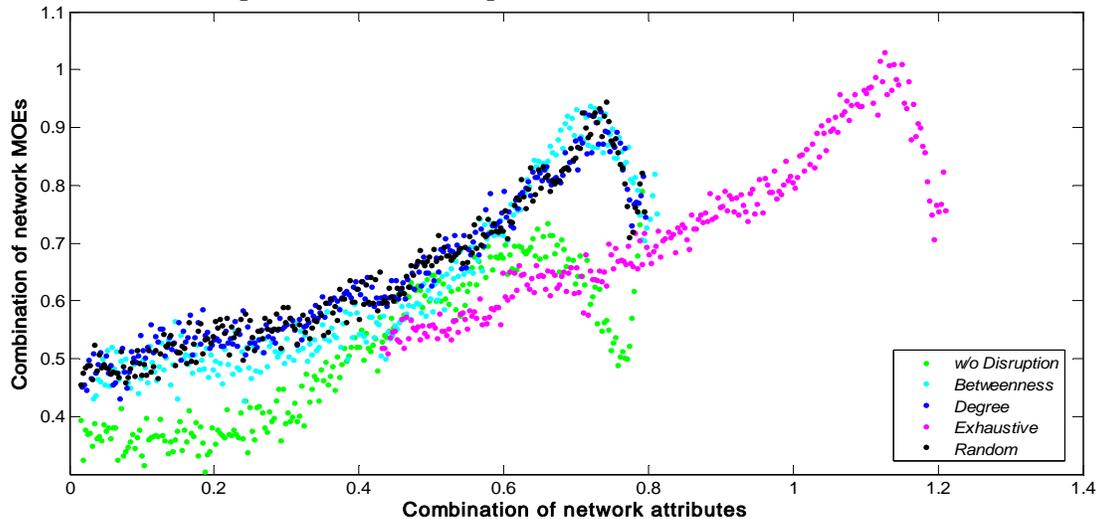
**Figure 2. Largest Eigenvalue after disruption vs. size of disruption for different algorithms**

Overall, the Exhaustive method dominated previously proposed methods in minimizing the largest eigenvalue and thus minimizing network connectivity. This dominance was significant, especially for smaller sizes of disruption. For disruption size equal to 15%, the exhaustive method reached its maximum performance and reduced the network connectivity by 50%. This means that those 15% of nodes are a subset of critical nodes. However, for disruption sizes less than 30%, the other methods (i.e., based on degree, betweenness, weighted betweenness and random) were not able to minimize the largest eigenvalue. More interestingly, the performance of the randomly-disrupting method was comparable to the performance of the previously developed methods (i.e., based on degree, betweenness) in minimizing connectivity.

To verify the presented results, we ran traffic simulation on the Guam network in five separate settings: (1) without disruption (i.e., the base case for comparison), (2) when 15% of nodes with highest degree were disrupted, (3) when 15% of nodes with highest betweenness were disrupted, (4) when 15% of nodes were randomly disrupted, and (5) when 15% of the nodes were selected based on the proposed method.

We used Canonical Correlation Analysis (CCA) to depict the simulation results of different methods (Borga, 2001). Figure 3 compares the traffic condition in the base case (i.e., without disruption, green dots) with traffic condition caused by

different methods. The Y-axis shows a linear combination of traffic density, queue and flow. The larger values of Y represent severe traffic conditions. Figure 3 shows a clear distinction between the results of the proposed method (pink dots) and all other results. While the curves have similar shape, there is an increase in Y for the proposed method. Thus, the proposed method was more effective in identifying the nodes whose disruption has severe impacts on traffic conditions.



**Figure 3. Simulation results for different methods plotted by CCA**

## DISCUSSION

The present study is an initial step towards developing a framework for identifying critical areas of a road network. The advantage of the proposed framework over existing ones may be its macroscopic viewpoint. This means that the framework focuses on network-wide connectivity rather than localized measures of importance (e.g., degree). However, the effectiveness of the proposed framework for some specific applications (e.g., emergency access to hospitals after disruption) needs further investigation.

One challenging aspect is that individuals will not be aware of precise location and duration of disruption during disruption period. As a result, some of them may travel using their habitual paths, which they used to travel. In addition, some of the individuals may use en-route information (e.g., radio type of information to receive updated news about disrupted areas). The DynusT model is able to run traffic simulation with the above assumptions. For example, during the disruption, one can assume that 60% of drivers will use the same paths as before disruption and 40% of drivers will use en-route information.

The practical applications may be grouped in two groups: pre and post disruption. Pre-disruption applications include: placing different monitoring devices (e.g., cameras) and road emergency recovery stations close to critical nodes/links. Post-disruption applications include starting road recovery actions from the critical nodes/links to minimize duration needed for connectivity improvement.

## CONCLUSION

The proposed eigenvalue-based approach performs better than existing criteria (e.g., degree and betweenness) in identifying a subset of critical nodes. The traffic simulation results verified that disrupting 15% of nodes, which were identified by the eigenvalue-based approach, led to critical traffic conditions compared to other existing methods. However, the proposed approach has some limitations. For example, we identified a subset of critical nodes without any constraints on their geographical closeness. In reality, the events that are caused by nature, such as floods, heavy snowfalls, and hurricanes may lead to disruption of multiple nodes/links in an area of the network. Future work might consider how to extend the proposed eigenvalue-based approach to identify critical areas (including multiple links) of a road network.

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