
Structural topologies by iterative multi-load dependent structural grammars and separate volume fraction topology optimisation

Hèrm Hofmeyer, h.hofmeyer@tue.nl

Eindhoven University of Technology, The Netherlands

Niels ten Heggeler, n.h.j.t.heggeler@student.tue.nl

Eindhoven University of Technology, The Netherlands

Abstract

For the development of a structural topology, structural grammars predefine locations of structural mass based on the building spatial design, whereas topology optimisation distributes structural mass freely and based on the loads. This paper presents a new method that involves first an iterative structural grammar that uses finite element simulations to optimise an initial structural topology for minimal structural mass. Hereafter topology optimisation is applied to modify the structural topology elements for minimal compliance. Due to the several groups of structural topology elements involved, a separate volume fraction is maintained for each group. Using an academic case study, the method is compared to standard structural grammars and topology optimisation, defining Pareto fronts for the objectives structural volume and strain energy. Future research will focus on the development of structural grammars that take into account openings, and on the implementation of the new method in simulations of co-evolutionary design processes.

Keywords: Structural Grammars, Topology Optimisation, Structural Design

1 Introduction

During a preliminary design phase, the two disciplines shaping a building the most are architecture, here limited to spatial design, and structural design and engineering. Spatial and structural design interact strongly too: Space can only be experienced if enclosed, e.g. by a structure, and more practically, spaces in a building can only exist if a structure allows these spaces to be used. The other way around: a stand-alone structure, e.g. an electricity pole, inevitably occupies space. This interaction is also such that a building representation for one of these disciplines can be seen as a design requirement for the other and vice versa in a co-evolutionary design process (Maher 2000), and designs can be classified for their spatial and structural determinedness (Hofmeyer 2007).

To support the complex activities involved with the design of a building, numerous research projects have been carried out, and dedicated design process models and support methods have been developed. Specifically for spatial and structural design, research has been carried out on integrative models (as part of design process models) and on optimisation methods and heuristic methods (as part of design support methods). *Integrative models* seek to model and describe the design process, taking into account the iterative and multidisciplinary design aspects. An important idea found in this research is that a design process is co-evolutionary, which means a design problem and solution interact (Maher 2000), in a dynamic environment (Gero 2004). *Optimisation methods* support the design process by finding optimal values for design variables in a problem defined by objective functions and boundary conditions: Form finding uses an inverse formulation of equilibrium to find tensile structures without bending (Bletzinger et al 2005), whereas structural optimisation works on a parametric model in which certain parameters are made optimal (Christensen & Klarbring 2009). Topology optimisation distributes relative density over a continuous finite element mesh to minimise the objective,

e.g. strain energy, which relates to stiffness (Bendsøe & Sigmund 2003, Stromberg et al 2011). *Heuristic methods* use domain knowledge, often condensed in rules, to generate possible design solutions. As an example, principles of physics of motions have been used to generate architectural layouts (Arvin and House 2002). Related to structural design, so-called expert systems have been developed to generate preliminary structural design concepts (Fenves 2000), recently also including assessments with finite element simulations (Steiner et al 2016). Shape grammars, which use geometrical design rules to generate spatial design layouts (Ruiz-Montiel et al 2014), can also be used to generate structural designs, e.g. truss structures (Shea & Cagan 1999), or frame-based halls (Geyer 2008). Here, these grammars will be referred to as "structural grammars".

In this paper two possible improvements for structural grammars are proposed. First, a grammar can be assisted during its generation process by information on the mechanical behaviour of the structural topology it proposes, section 2. Second, the results of a grammar can be improved by using topology optimisation modified to work with structural topologies, section 3. Using an academic case study in section 4, the proposed improvements will be compared with existing grammars being not improved by topology optimisation. Finally, conclusions will be given in section 5.

2 Iterative multi-load dependent structural grammars

2.1 Initialisation

The starting point here is a building spatial design, which is a set of orthogonal spaces, figure 1. Each space is given by the dimensions width (in x-direction), depth (in y), and height (in z), and the x-y-z-position of the corner nearest to the origin. Furthermore, in this research a structural grammar 0, 1, 2, 3, or 4 (to be explained below and in section 4) is assigned to each space individually. Assuming here that all spaces are assigned grammar 3, which is the iterative multi-load dependent structural grammar, this grammar adds a structural topology wall/slab to each vertical space surface and the top horizontal space surface. As these topology elements are the result of grammar 3, they are assigned a so-called type number 3 too, which indicates these walls or slabs may be replaced by other structural topology elements during the iteration. After application of the grammar, possible coincident elements are handled by deleting first trusses having two keypoints in common with a beam, then deleting trusses having two keypoints in common with a wall/slab (so including diagonals), then the same for beams, and finally for coincident wall/slab elements, where only the one with the highest type number is kept.

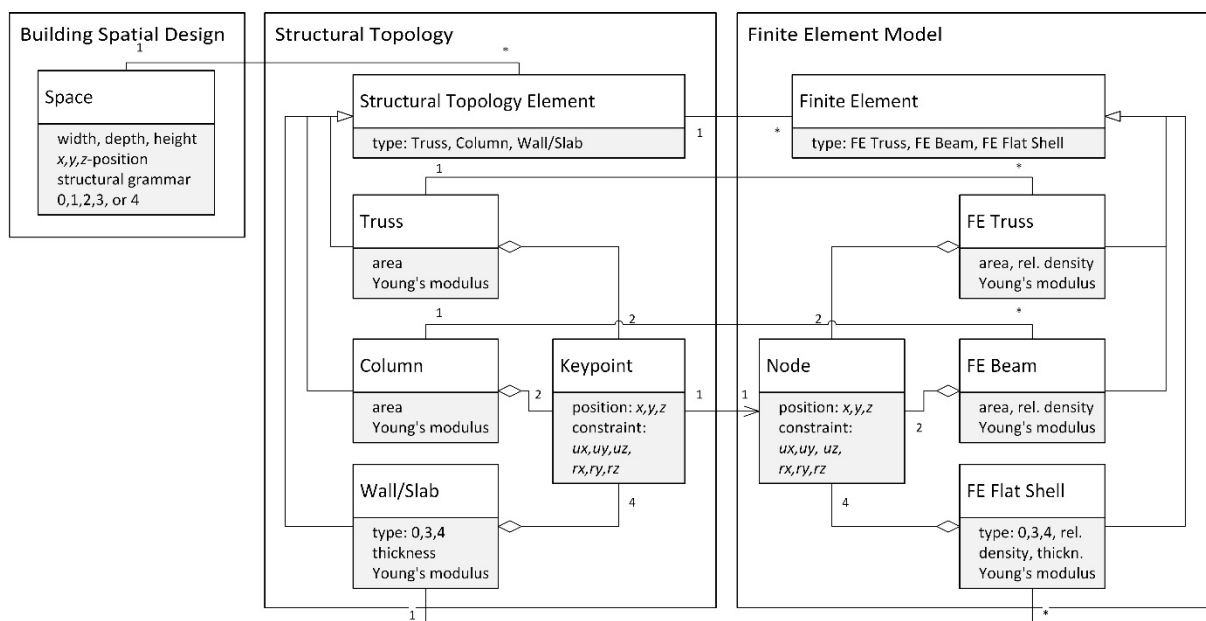


Figure 1 Data model presents the building spatial design, the structural topology to be designed, and the finite element model that assesses the topology's mechanical behaviour

For predicting the mechanical behaviour of this initial topology, a finite element model is developed. First, constraints for zero displacements in x, y , and z -direction are applied to all keypoints that share the lowest occurring z -coordinate. Then every structural topology element (a wall/slab here) is meshed with a single four node finite element FE Flat Shell. For in-plane behaviour, a standard formulation is used as found in Cook et al (2002) and for bending behaviour the DKQ formulation is used (Batoz & Tahar 1982). Drilling stiffness is accounted for by taking for each term the average of all terms in the stiffness matrix. The FE Flat Shell has a thickness equal to 150 mm, a Young's modulus is taken equal to 30.000 N/mm², and Poisson's ratio is 0.3. Five different load cases can be distinguished: a live load, and 4 wind loads directed in either positive or negative x , or y -direction. In the implementation, normally a load case is applied to the building spatial design, then transferred to the "underlying" structural topology, and finally applied as nodal loads in the finite element model (Hofmeyer & Davila Delgado 2015). However, here simplified load cases are directly applied to the finite element model. This to enable exact comparisons between the several structural grammars and independent of the mesh size, as presented in section 4. As such, finite element nodes are selected for their (real) x, y , and z -coordinates yielding a remainder 0 when divided by a (real) number that represents a grid size used for the building spatial design. Selected nodes are then loaded with 1000 N in z -direction for live load, and with 1000 N in either positive or negative x - or y -direction for the wind load cases. The finite element model is assembled, and hereafter solved using the SimplicialLDLT solver in Eigen, a C++ template library for linear algebra (Guennebaud & Benoit, 2016).

2.2 Iterative procedure

The iterative procedure starts with assigning a type number 3 to each FE Flat Shell that has the same nodal coordinates as the keypoint coordinates of a wall/slab with type number 3. As such, the 4 stiffness matrices for each FE Flat Shell of type 3 are multiplied with 1e-06 to ensure the element stiffness will not influence behaviour of finite elements representing already replaced walls/slabs for the final design (having normal stiffness). Note that this multiplication concerns 4 matrices because separate stiffness matrices are derived for axial action only, bending only, shear only, and total action. For axial action or shear action these are found by using a reduced constitutive matrix in the standard formulation for in-plane behaviour as shown in equation (1), by the second and third matrix respectively.

$$\frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{E}{1-\nu^2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \quad (1)$$

For bending action only, the DKQ formulation is used without adding the in-plane behaviour. Then the finite element simulation as presented above is carried out again. After the simulation the strain energy due to axial action only, bending only, shear only, and total action, summed over all load cases, is calculated for each type 3 FE Flat Shell. Similarly, for each FE Flat Shell of another type, the strain energy is determined, but now only for total action. Using this data, the strain energy over all load cases is calculated summed up for either all type 3 or all other type FE Flat Shells.

After ranking the type 3 FE Flat Shells for their shear strain energy only, the FE Flat Shell with the highest shear energy -if still above a certain threshold (here 100.000 Nmm)- is deleted and replaced by six new FE Trusses, one along each FE Flat Shell edge and two diagonal ones across the FE Flat Shell.

As new finite elements have been generated, a check for coincident elements is carried out, which results in no coinciding FE Trusses, FE Beams, and FE Flat Shells, preferring a FE Flat Shell to a FE Beam to a FE Truss. However FE Trusses are allowed to coincide with the low stiffness type 3 Flat Shell elements to avoid the later to be replaced. More details can be found in Ten Heggeler (2016). After all newly generated FE Flat Shells (for a certain approach not followed here) have been set to type 0, all FE Truss, Beam, and Flat Shells (the later of all

types) are transformed to their corresponding structural topology elements (see figure 1), actually representing a new structural topology.

This structural topology is subject to a next loop of the iterative procedure. Boundary constraints (modelling the foundation) are applied as explained in section 2.1. Wall/slab structural topology elements are meshed (see section 2.1 as well), and each truss (cross-section 300×300 mm², Young's modulus = 30000 N/mm²) is meshed by a single standard two node linear truss element, each node having the 3 degrees of freedom u_x , u_y , and u_z (Cook 2002), and a beam with the same cross-section is modelled by a single standard two node beam element for which each node has 6 degrees of freedom, namely u_x , u_y , u_z , r_x , r_y , and r_z (Przemieniecki 1968). After the definition of the loads (section 2.1), the iterative procedure continues as explained at the beginning of this section 2.2. If after ranking the type 3 FE Flat Shells for their shear strain energy only, the FE Flat Shell with the highest shear energy is below the threshold (in this paper 100.000 Nmm), the iterative procedure is successfully finished.

3 Separate volume fraction topology optimisation

Commonly, topology optimisation operates on a finite element model with a continuum of elements of a single formulation and similar size. However, structural topologies consist of trusses, beams, and wall/slabs, and are represented by finite elements having different formulations and sizes. Therefore, in this paper an alternative approach is used, using separate volume fractions and settings for each of the following 4 groups of finite elements: FE flat shell type 0; FE flat shell type 4, FE Beam and FE Truss (see figure 1). Using as a basis the sensitivity filtering based approach described by Sigmund (2001), the optimisation problem is defined as follows:

$$\min_{\mathbf{x}_w, \mathbf{x}_f, \mathbf{x}_b, \mathbf{x}_t} \{c_w(\mathbf{x}_w) + c_f(\mathbf{x}_f) + c_b(\mathbf{x}_b) + c_t(\mathbf{x}_t)\} = \mathbf{u}^T \mathbf{K} \mathbf{u} = \sum_{e=1}^n \mathbf{u}_e^T (x_e)^{p_e} \mathbf{K}_e \mathbf{u}_e \quad (2)$$

$$\frac{V_w(\mathbf{x}_w)}{V_{w0}} = \frac{V_f(\mathbf{x}_f)}{V_{f0}} = \frac{V_b(\mathbf{x}_b)}{V_{b0}} = \frac{V_t(\mathbf{x}_t)}{V_{t0}} = f \quad (3)$$

$$0 < x_{\min} \leq x_e \leq 1 \quad (4)$$

$$\mathbf{K} \mathbf{u} = \mathbf{f} \quad (5)$$

In equations (2) to (5), the objective is to minimise the compliance c , which is a function of the weighted strain energy over all elements, the latter expressed as a function of the global displacement vector \mathbf{u} and global stiffness matrix \mathbf{K} . In the objective function, e is a finite element identifier, n the total number of elements, and x_e is the relative density of element e , where relative densities are stored for all FE flat shell type 0 in vector \mathbf{x}_w (w stands for wall/slab), for all FE flat shells type 4 in vector \mathbf{x}_f (f stands for flat shell), and for all beams and trusses in vector \mathbf{x}_b and \mathbf{x}_t respectively. The variable p_e is a penalisation factor, which may be different for each group of elements, \mathbf{u}_e is the displacement vector of an element and \mathbf{K}_e is an element's stiffness matrix. The constraints in equation (3) keep the ratios between the structural volumes (being functions of the densities) $V(\mathbf{x})$ and the initial volumes V_0 constant to a user selected value f . Equation (4) ensures that the densities are kept between a maximum of 1 and a minimum x_{\min} , which is slightly larger than zero to avoid singularities of the stiffness matrix. Finally, the finite element formulation is expressed in equation (5), where \mathbf{f} is the force vector. The optimisation is initiated by computing the modified stiffness matrices of the elements using initial column vectors $\mathbf{x}_{w,f,b,t}$ as shown in equation 6. For FE Flat Shell type 4 elements, the commonly used value for $p_e = 3$ is used, whereas for FE Flat Shell type 0, FE Beam, and FE Truss elements $p_e = 1$ values are used, as these lead to a more realistic material distribution, for more details see Ten Heggeler (2016).

$$\forall_e \left\{ \mathbf{K}_e \rightarrow x_e^{p_e} \mathbf{K}_e \right\} \quad (6)$$

Using these modified element stiffness matrices, the global stiffness matrix is assembled and the corresponding equilibrium equations $\mathbf{Ku}=\mathbf{f}$ are solved for the displacements \mathbf{u} . As such, the compliances $c_w(\mathbf{x}_w), c_f(\mathbf{x}_f), c_b(\mathbf{x}_b)$, and $c_t(\mathbf{x}_t)$ can be calculated using equation (2). Using the latter equation (2) again, the sensitivity of the compliances to the variation of an element's density is calculated as described by equation (7), using for elements e only the elements belonging to the group w, f, b , or t .

$$\frac{\partial c(\mathbf{x}_{w,f,b,t})}{\partial x_{e \in \{w,f,b,t\}}} = -p_e x_e^{(p_e-1)} \mathbf{u}_e^T \mathbf{K}_e \mathbf{u}_e \quad (7)$$

For wall/slabs that are assigned to be optimised by topology optimisation over their area, consequently each modelled with elements of the FE flat shell type 4 group, a sensitivity filter is applied to avoid checkerboard patterns. This filter modifies the above sensitivity of each finite element as a function of the sensitivities of neighbouring elements, taking into account the density and distance of these elements:

$$\frac{\hat{\partial c}_f(\mathbf{x}_f)}{\partial x_{e \in f}} = \frac{1}{\sum_{i \in f} x_e H_{ei}} \sum_{i \in f} \left\{ x_i H_{ei} \frac{\partial c(\mathbf{x}_f)}{\partial x_i} \right\} \quad (8)$$

$$H_{ei} = \max(0, r_{\min} - \text{dist}(e, i)) \quad (9)$$

Note that for the other groups of finite elements no sensitivity filter is applied. This because first it would make no sense to average densities over different structural topology elements (in the case of trusses meshed by 1 finite element only to avoid kinematic mechanics, or wall/slabs assigned to be optimised by topology optimisation for their -uniform over their area-thickness and consequently meshed by only 1 finite element too). Second, for beams, which still may be meshed by several finite elements, checkerboard patterns do not occur. For each group of finite elements separately (if the compliance of that group is larger than a certain threshold, here 0.001), new densities x_e are calculated by means of an optimality criteria method as shown below in equations (10) and (11).

$$\forall_{i=w,f,b,t} \left\{ x_e^{new} = x_e \sqrt{\frac{-\hat{\partial c}_i(\mathbf{x}_i) / \partial x_e}{\lambda_{w,f,b,t} (\partial V_i(\mathbf{x}_i) / \partial x_e)}} \right\} \quad (10)$$

$$x_e^{new} \geq \max(x_{\min}, x_e - m) \quad \wedge \quad x_e^{new} \leq \min(1, x_e + m) \quad (11)$$

The change in relative density is limited to a value m to ensure smooth changes in the topology, here set to 0.2. The total sum of all densities of a group is scaled to equal the design volume fraction f in equation (3) by a bi-sectioning algorithm that finds Lagrangian multiplier λ . As in this paper the approach is such that practically finite elements in a specific group have all the same volume (with only a slight difference for diagonal trusses compared to vertical or horizontal trusses meshed with the same seed), volume sensitivity is not taken into account, similar to Sigmund (2001). With for each group the new density vector now predicted, the change of this vector compared to the previous attempt can be calculated. For the groups of FE Beams and FE Trusses this change is artificially increased, as it was found (Ten Heggeler 2016), that for these groups more iterations are needed for convergence, given by equation (12).

$$\Delta \mathbf{x} = \max \left(\left\| \mathbf{x}_w^{new} - \mathbf{x}_w \right\|_{\infty}, \left\| \mathbf{x}_f^{new} - \mathbf{x}_f \right\|_{\infty}, 10 \left\| \mathbf{x}_b^{new} - \mathbf{x}_b \right\|_{\infty}, 10 \left\| \mathbf{x}_t^{new} - \mathbf{x}_t \right\|_{\infty} \right) < 0.02 \quad (12)$$

The strain energy for the optimised structural topology is calculated with the same finite element simulation, however, where now for each element the density is taken into account, without any penalisation factor. Finally, the relative density of a flat-shell element scales the normal and the bending *stiffness* with the same order, see eq. (6). In this paper this scaling is defined as "optimizing the *thickness*" to illustrate its practical implication. However, note that strictly seen this is incorrect as for a flat shell, thickness scales linearly with normal stiffness and cubically with bending stiffness.

4 Academic case study

4.1 Application

To demonstrate the iterative multi-load dependent structural grammar and separate volume fraction topology optimisation, and to compare these with other grammars, an academic case study is presented as shown in figure 2. The building spatial design is an orthogonal building with an archway. To the left and right of the archway four spaces, each 3x3x3 m. and on two levels, support the upper part of the building. Above the archway two levels are present, each consisting of 2x4 spaces, which dimensions are also 3x3x3 m. The process to be followed is shown in figure 3.

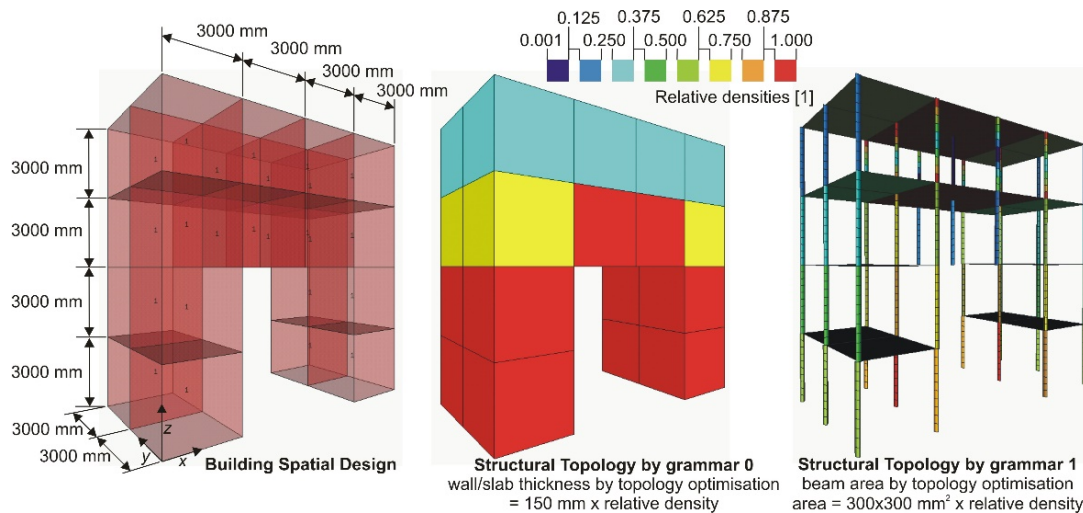


Figure 2 Building spatial design and structural topologies with optimised element properties

First, for all spaces so-called structural grammar 0 is used. This grammar adds a structural topology element wall/slab to each vertical surface and to the horizontal top surface of each space. Each element is assigned a Young's modulus of 30000 N/mm², a Poisson's ratio of 0.3, and has a thickness of 150 mm. As these structural topology elements are the result of grammar 0, they are assigned a so-called type number 0. Type 0 indicates that for topology optimisation these walls or slabs are optimised for their thickness only, and consequently will be meshed by 1 finite element there. As presented in section 2.1, coincident elements are removed and the loading and boundary constraints are formulated. In this case study for loading only the live load case is used, resulting in a downward force on each keypoint of the structural topology equal to 1000 N.

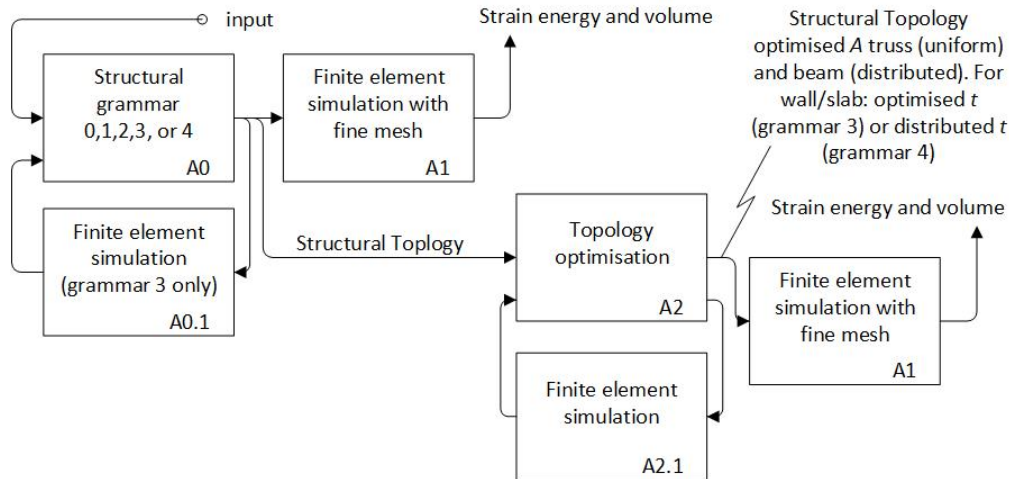


Figure 3 Structural grammar followed by topology optimisation, fine mesh simulations for energy determination

The performance of the structural topology is assessed by a finite element model. In this model, every structural topology element is meshed by 10×10 FE Flat Shell elements, their formulation described in section 2. After solving the finite element linear system of equations, the strain energy over all load cases (here only 1) and all elements, and their total volume are calculated as shown in table 1. Now, separate volume fraction topology optimisation is applied on the structural topology, as explained in section 4, using volume fractions f equal to 0.8, 0.5, and 0.3. As all wall/slab elements are type 0, only their thickness will be subject to optimisation, and therefore in the finite element model needed for topology optimisation, each wall/slab element is meshed with a single FE Flat Shell element. The output of the topology optimisation is a relative density for each FE Flat Shell element, and thus for each wall/slab, as shown in figure 2 in the middle. Finally, the performance of the optimised structural topology is assessed by a finite element model with a finer mesh (10×10 FE Flat Shell elements for each wall/slab) and results for strain energy are calculated normally, without a penalty as used in the topology optimisation process, and as found in table 1.

Table 1 Strain energy and structural volume as function of grammar and volume fraction f for optimisation

Grammar [1]	f TO [1]	Volume [m ³]	Strain energy [Nmm]			
			FE Flat Shell + FE Beam + FE Truss	FE Flat Shell	FE Beam	
0	no TO	130	98	98		
wall/slab,	0.8	104	99	99		
opt. uniform t	0.5	65	116	116		
wall/slab	0.3	39	181	181		
1	no TO	47	1105	728	377	
columns + slab,	0.8	38	1125	729	396	
opt. A beam	0.5	24	1412	817	595	
and uniform t slab	0.3	14	2249	1221	1028	
2	no TO	102	191	17	174	
trusses + slab,	0.8	82	194	17	177	
opt. A truss	0.5	51	221	10	211	
and uniform t slab	0.3	31	288	3	285	
3	no TO	58	310	1	293	16
replaces wall/slab	0.8	46	311	0	292	19
opt. A tr.+beam	0.5	29	368	1	340	27
and						
uniform t wall/slab	0.3	17	596	1	550	45
4	no TO	130	98	98		
wall/slab,	0.8	104	98	98		
opt. area	0.5	65	112	112		
distributed						

<i>t</i> wall/slab	0.3	39	158	158
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Secondly, grammar 1 is applied, which adds 4 columns, one on each vertical edge of the space, and which support an added slab type 0 located at the top surface of the space. The columns have a cross-section equal to 300×300 mm and use a Young's modulus of 30000 N/mm² and a Poisson's ratio of 0.3. For the slab the same properties are used as in grammar 0. Following the same procedure as for grammar 0, results are shown in table 1 and figure 2 on the complete right. Grammar 2 generates 8 trusses, one at each vertical edge of a space and 4 diagonally across each vertical surface of a space, again supporting an added slab (type 0). The trusses have a cross-section equal to 300×300 mm and use a Young's modulus of 30000 N/mm². Results are again shown in table 1 and figure 4 top left.

Application of the iterative multi-load dependent structural grammar 3 follows section 2 and is illustrated in figure 5. After finishing successfully, all remaining type 3 FE Flat Shells are deleted and replaced by 4 circumferential FE Beams; horizontal diagonal FE Trusses are deleted; and for all horizontal wall/slabs not type 4 a FE Flat Shell type 0 is generated, followed by the deletion of coincident elements as explained. For this structural topology, strain energies are determined with a finely meshed finite element model as described for the other grammars. Then finally separate volume fraction topology optimisation is carried out, with results as listed in table 1 and shown in figure 4 top right.

Structural grammar 4 starts equal to grammar 0, however all walls and slabs obtain type 4, which means that topology optimisation is carried out on walls and slabs meshed each by 10×10 finite elements. The resulting optimal relative density per element may still be regarded as a scaling of the thickness, although now varying over the area of the wall or slab. Results are presented at the bottom rows of table 1 and figure 4 bottom left. Note that this last figure also gives an impression of the fine mesh used to assess the total strain energy of the other grammars.

4.2 Results and discussion

Comparing the grammars for total strain energy using no topology optimisation in table 1, it is clear that grammar 0 makes the building spatial design most stiff. This by generating a wall or slab for every surface of each space. Using topology optimisation using a volume fraction of 0.8 does not change the relative density distribution visually and strain energy does not increase.

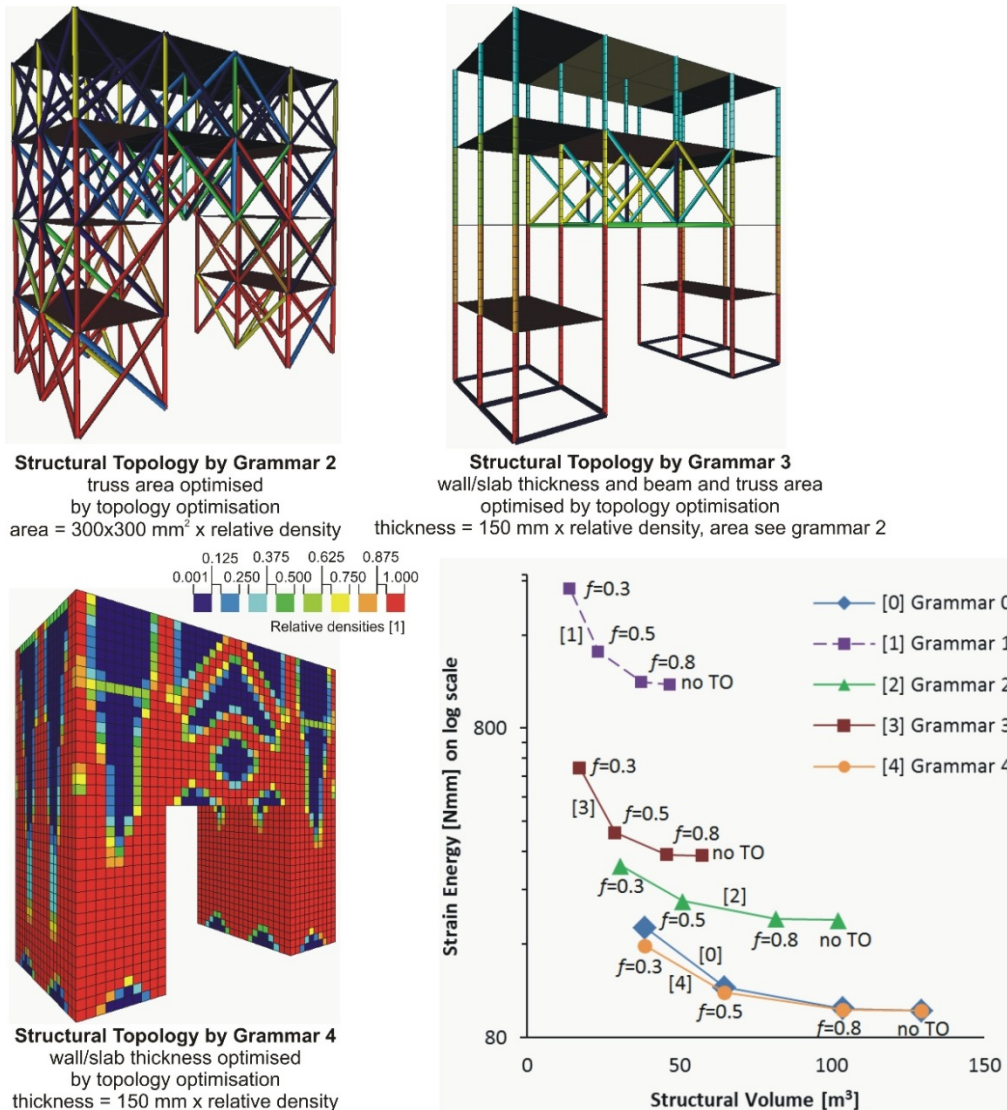


Figure 4 Structural topologies generated by grammar 2 to 4 and Pareto fronts for all grammars bottom right

For further reduction in volume, e.g. $f=0.5$, topology optimisation strengthens the support and bridging parts of the building as shown in figure 2 in the middle. Generating beams and slabs as carried out by grammar 1 naturally results in a lower volume than using walls and slabs only. However, total strain energy increases significantly: in the building bending occurs due to the archway, and this overall bending is distributed via ineffective local bending of FE Beams and FE Flat Shells. Again, for a first reduction of volume in topology optimisation, almost no change in total strain energy is found, which is also the case for all grammars further to be discussed. For larger volume reductions, table 1 shows the FE Beams take increasingly more strain energy from the FE Flat Shells. This can be understood as the multiple FE Beams allow for a relative density distribution over the length of a single structural topology column, whereas the single FE Flat Shell for every structural topology slab allows only for a single relative density for each slab. In other words, beams are more part of topology optimisation than the slabs. As the effectiveness of topology optimisation decreases when the volume fraction is reduced (see the Pareto fronts in figure 4), the effectiveness of the FE Beams also decreases, and more than the FE flat shells.

Grammar 2, which generates vertical and diagonal trusses and a slab, does not reduce the structural volume of grammar 0 so strongly, which can be explained by the application of two horizontal trusses and two diagonal trusses all with a cross-section of 300x300 mm² for each vertical surface of the space. Total strain energy is much lower than for grammar 1, as the diagonal struts distribute the bending in the building by far better than the beams. As the

slabs are not bent at their nodes by the trusses, which was the case in grammar 1 by the beams, they hardly display strain energy.

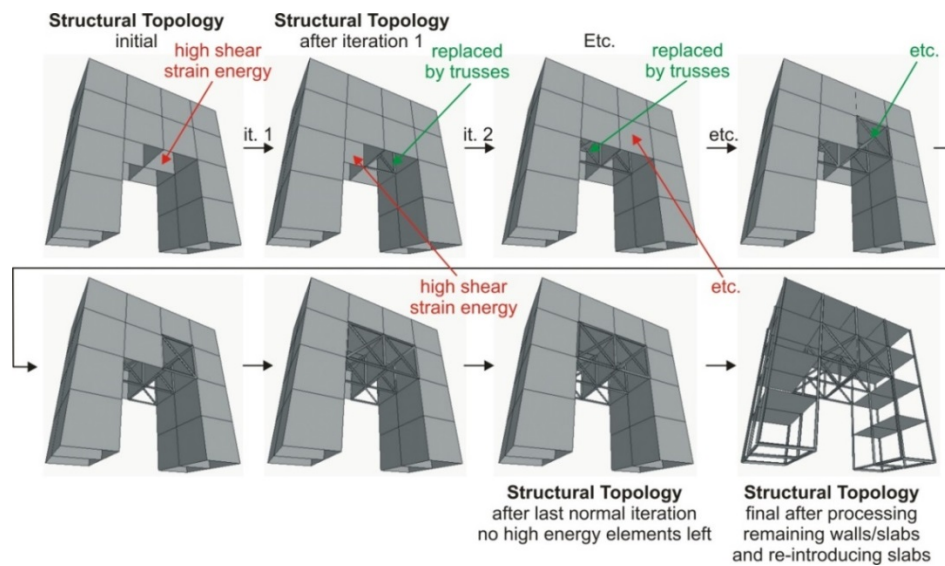


Figure 5 Iterative multi-load dependent structural grammar

Grammars 0, 1, and 2 use heuristic knowledge to position structural topology elements, which are then only fine-tuned by topology optimisation with respect to their dimensions. Grammar 3 is different as here the heuristic knowledge is supported by predictions of the mechanical behaviour in an iterative way. By the specific set-up of the grammar in this paper as explained in section 2, the generated structural topology is similar to topologies created by grammar 2 and 3: Namely in surfaces where shear strain is high the diagonals of grammar 2 can be found, where shear strain is less the beams of grammar 3 can be seen instead. Due to this effective application of diagonals, the total strain energy of a structural topology by grammar 3 is more similar to the energy due to grammar 2 than grammar 1, the latter having a much higher strain energy. Besides, using beams where this is possible, instead of trusses which involved a large volume as explained above, reduces the volume to a level near to grammar 1. As a result, the iterative grammar combines the low strain energy of grammar 2 with the low structural volume of grammar 1. For topology optimisation, like grammar 1, the iterative grammar FE Beams show a relative increasing contribution to the strain energy compared to FE Flat Shells, and the same explanation can be given as above.

The last grammar 4 still uses heuristic knowledge by allocating walls and a slab at each space its boundaries, but topology optimisation then takes over to at least theoretically distribute the relative densities such that trusses or beams and voids can occur, as shown in figure 4 bottom left. Although promising as such, strain energy is not reduced significantly compared to grammar 0.

The Pareto fronts in figure 4 bottom right confirm that (a) topology optimisation is not strongly effective for structural topologies of walls and slabs as shown by grammar 0 and 4; (b) the other grammars may reduce the volume but inevitably with increased strain energy; (c) the iterative grammar 3 produces structural volumes almost as good as grammar 1 and a related strain energy almost as good as grammar 2; (d) also, the fronts show more clearly than table 1 that for decreasing prescribed structural volume, strain energy increases in a nonlinear fashion.

5 Conclusions

A structural grammar, which adds structural topology elements to a building spatial design, has been extended by a procedure that modifies elements iteratively, based on the topology's mechanical behaviour.

To fine tune the structural topologies found by a structural grammar, topology optimisation has been modified to be able to optimise problems with several types of finite elements, not necessarily forming a continuum.

Iterative multi-load dependent structural grammars are useful in optimising a structural topology, as their improvement easily matches that of topology optimisation (compare grammar 3 to 1 in figure 4 bottom right) and amends the heuristic principles of the grammars with feedback of mechanical behaviour.

The case study has shown that different from standard topology optimisation, separate volume fraction topology optimisation enables the optimisation of problems with several types of finite elements, not necessarily forming a continuum.

Future research will focus on the development of structural grammars that take into account openings, and on the implementation of the new method in simulations of co-evolutionary design processes.

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